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AN ATTEMPT TO DETECT A MAGNETIC FIELD AS THE RESULT OF THE ROTATION OF A COPPER SPHERE AT HIGH SPEED.

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Introduction: One type of magnetic field attributable to rotation is well known from the investigations of S. J. Barnett. It arises from the gyroscopic action of the rotation of the electron orbits of the atom which are responsible for the magnetic properties of the substance, and is zero when the intrinsic magnetic moment of the atom is zero. We shall refer to this effect as the Barnett effect. It is not this effect with which we are concerned in the investigation to be described, but rather with the possibility of the creation of a magnetic field as a result of causes of even more subtle nature, causes not included in our scheme of classical electrodynamics.

The possibility of producing a magnetic field by pure rotation of neutral matter finds one of its chief fields of interest in connection with attempts to explain the origin of the

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earth's magnetic field. The Barnett effect is inadequate for this purpose even though we should assume the earth to be made entirely of iron, for it would account for a magnetic field only 10^{-10} of the earth's magnetic field.

Almost any theory of the origin of a magnetic field as the result of rotation of a neutral sphere makes the field depend upon the density of the material, the angular velocity, and the radius of the sphere. In a recent paper [1] it has been shown by one of us that, if we confine ourselves to an expression of the magnetic field involving only the product of simple powers of the angular velocity and radius, knowledge of the magnetic fields of the earth and sun, and of the fact that the magnetic field of a small body rotating at high but attainable speed is not so large as to have forced itself upon our observations long ago, are sufficient to limit us to two possible expressions for the magnetic field at the surface of the sphere. According to one of them, the field at the surface is proportional to $D\omega a^2$ and according to the other it is proportional to $D\omega^4 a^4$, where D is the density, ω the angular velocity, and a the radius of the sphere. The values predicted by these two possibilities for the magnetic field H_z at the pole of the Sun and at the pole of a copper sphere 10 cms. in radius rotating 200 times per second, are given in Table 1,

TABLE I

Current Density Proportional to	Form of H_z	H_z Gauss		
		Sun	Earth	Copper Sphere 10 cms. in Radius, Rotating at 200 revs. per sec.
$D\omega r$	$AD\omega a^2$	58	0.5	3.2×10^{-9}
$D\omega^3 r^4$	$AD\omega^4 a^4$	42	0.5	4.4×10^{-3}

A being a constant. The constant of proportionality A is adjusted to give the value 0.5 gauss at the earth's pole. The detailed development of these possibilities present the matter in a light in which the sphere acts as though currents circu-

[1] W. F. G. Swann, "A Generalization of Electrodynamics, Consistent with Restricted Relativity and Affording a Possible Explanation of the Earth's Magnetic and Gravitational Fields, and the Maintenance of the Earth's Charge," *Phil. Mag.*, S. 7, Vol. 3, pp. 1088–1136, 1927.

lated about the axis of rotation. The apparent current density depends upon the angular velocity and the distance from the axis of rotation. Corresponding to the first of the possibilities cited, the apparent current density i is proportional to $D\omega r$ and according to the second, it is proportional to $Dr^3\omega^4$.

The former of these possibilities would give for the small sphere a field far below the limits of measurement while the latter gives a value which is measurable, though with some difficulty. Since it is not practicable to measure the field at the surface, use was made of the expression from which the field 4.4×10^{-3} given at the surface was obtained, for calculating the field at places in the vicinity of the sphere where it could be measured.

The expression for the magnetic potential Ω in electromagnetic units, and due to the sphere, is[2]

$$\Omega = \frac{-8}{35C} A\pi D\omega^4 a^4 \left\{ \frac{2}{3} P_1(\theta) \frac{a^3}{R^2} - \frac{1}{9} P_3(\theta) \frac{a^5}{R^4} \right\}, \qquad (1)$$

in which the current density has been written as $i = AD\omega^4 r^3$, where a is the radius of the sphere, R the radius vector to the point at which Ω is sought, θ is the angle made by this radius vector with the axis of rotation, and the P's are the Legendre polynomials. C is the velocity of light.

The values of the magnetic fields, H_R parallel to the radius vector, H_θ perpendicular to the radius vector, and H_r perpendicular to the axis of rotation, are, respectively:

$$H_R = \frac{-32}{35C} A\pi D\omega^4 a^4 \left\{ \frac{1}{3} \frac{a^3}{R^3} P_1(\theta) - \frac{1}{9} \frac{a^5}{R^5} P_3(\theta) \right\}, \qquad (2)$$

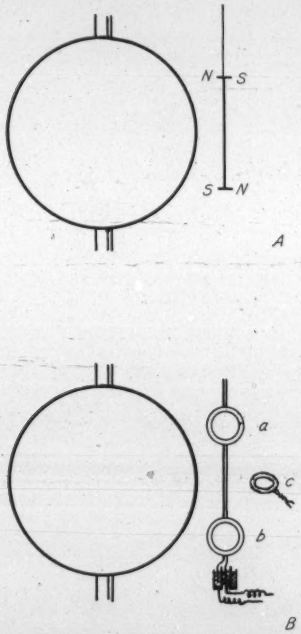
$$H_\theta = \frac{-8}{35C} A\pi D\omega^4 a^4 \left\{ \frac{2}{3} \frac{a^3}{R^3} \sin\theta - \frac{5}{9} \frac{a^5}{R^5} P_2(\theta) \sin\theta \right\}, \qquad (3)$$

$$H_r = \frac{-8}{105C} A\pi D\omega^4 a^4 \times \left\{ 6 \frac{a^3}{R^3} \cos\theta \sin\theta - \frac{1}{3} \frac{a^5}{R^5} (3\cos\theta + 7P_3(\theta)) \sin\theta \right\}. \qquad (4)$$

[2] See "A Generalization of Electrodynamics, etc.," *Phil. Mag.*, S. 7, Vol. 3, pp. 1088–1136, 1927. By replacing I by $AD\omega^4\sigma^3 \sin^3\beta\sigma d\beta d\sigma$ in the first expression on p. 1116 of this paper, and then integrating from $\sigma = 0$ to $\sigma = a$, and from $\beta = 0$ to π, equation (1) given above will readily be obtained.

Experimental Arrangement: For the detection of the magnetic field, an astatic magnetometer is the instrument which naturally suggests itself, the two magnets of the astatic system being supported symmetrically with respect to the equator of the sphere as shown in Fig. 1, *A*. The sphere would then act in an additive manner on the two elements of the astatic system; while the system would be astatic as regards uniform magnetic fields such as that of the earth. This arrangement was used in preliminary experiments; but, with a copper sphere 10 cms. in radius, rotating 200 times per

FIG. 1.



second, it is difficult to reduce vibrations to an extent which renders practicable the use of a system of this kind. For this

reason, we resorted to an astatic earth inductor compass, arranged as in Fig. 1, *B*. The coils *ab* rotated about a vertical axis. They were wound in opposite directions, so that they were additive in their effects as regards the magnetic field of the sphere, but subtractive as regards any uniform field such as that of the earth. The coils were connected through mercury contacts to an amplifier, which operated a Leeds & Northrup alternating current galvanometer. The current to the field coils of the galvanometer was supplied by a small generator which was driven on the same shaft as the earth-inductor compass, but at a distance of about 2 m. from the compass. The axis of the coils was placed as near as possible to the rotating sphere; and, the distance between the coils was then chosen so that the value of H_r calculated for the center of one of the coils was a maximum.

In order to standardize the readings of the galvanometer, use was made of a small coil *C*, Fig. 1, *B*, of radius 1 cm., and with 18 turns, situated with its plane horizontal and its center in the plane of the axes of the sphere and compass and at a distance of 2.8 cms. from the axis of the compass, on the side remote from the sphere. The horizontal field component due to such a coil is:

$$H_r = \frac{3\pi N}{20} I \frac{b^2}{R^3} \sin 2\theta \left[1 - \frac{5}{8} \frac{b^2}{R^2} (7 \cos^2 \theta - 3) \right],$$

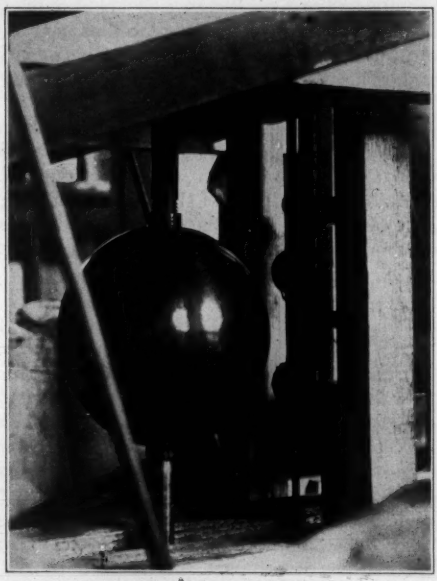
where I, the current in the coil is in amperes, N is the number of turns, b is the radius of the coil, R the radius vector to the point at which the field is derived, and θ the angle made by the radius vector with the axis of the coil. The galvanometer deflection due to a known current in this coil served to determine the field at the center of one of the earth-inductor coils, and so provided a means of reducing to absolute measure the galvanometer deflections obtained in the main experiments.

The earth-inductor compass was rotated by a belt and pulley attached to a small motor situated at a distance of about 5 m. from the sphere, and the sphere itself was rotated by means of a friction drive made of non-magnetic parts, and driven by a 2.5 horsepower motor situated at a distance of 5 m. from the sphere. The plan adopted was to spin the sphere up to speed, disconnect it from the friction drive by

means of a clutch, stop the motor, and then take readings as the sphere died down in speed.

Quite apart from any magnetic field of the type sought in this investigation, we have, of course, a magnetic field arising from Foucault currents generated by the sphere rotating in the earth's magnetic field. By considering the nature of the Foucault currents generated in a rotating sphere,[3] it is easy to see that such currents would cancel as regards

FIG. 2.



their magnetic effects on the earth-inductor compass in the case of a sphere rotating in a uniform field, and with the center of the compass in the equatorial plane of the rotation.

[3] J. J. Thomson, "Recent Researches in Electricity and Magnetism," pp. 546–557.

It is, however, impracticable to produce and maintain with certainty a condition of symmetry satisfactory for compensation in this manner, and, consequently the sphere was surrounded by a pair of Helmholtz coils which were adjusted so as to annul the earth's field.

FIG. 3.

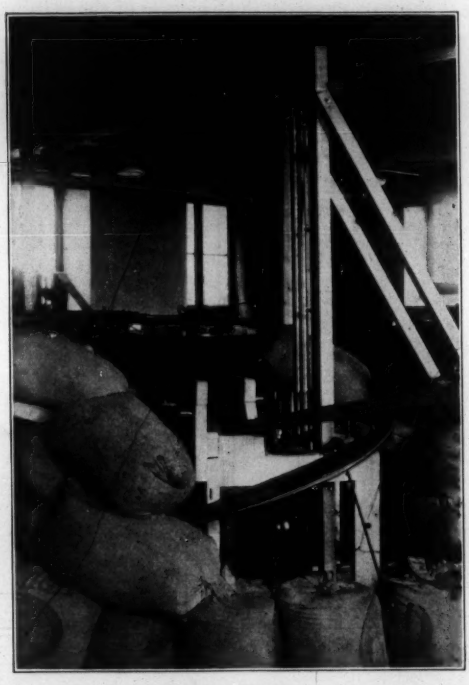
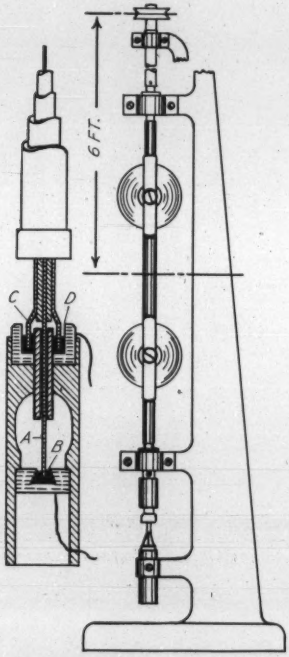


Fig. 2 shows the copper sphere, 10 cms. in radius, which was used in the experiment. It was necessary to balance it very carefully, before use. Fig. 3 is a picture of the sphere with its friction drive mounted in position with the Helmholtz

coils. The sand bags were not an unnecessary precaution when it is realized that the sphere increased in equatorial diameter by 2.4 mm. after rotation to a speed of 200 revolutions per second. The wooden frame shown in the upper portion of Fig. 3 carries the shaft of the earth-inductor compass. The coils of the compass itself are not visible in the picture.

Fig. 4 shows a more detailed picture of the coils and com-

FIG. 4.

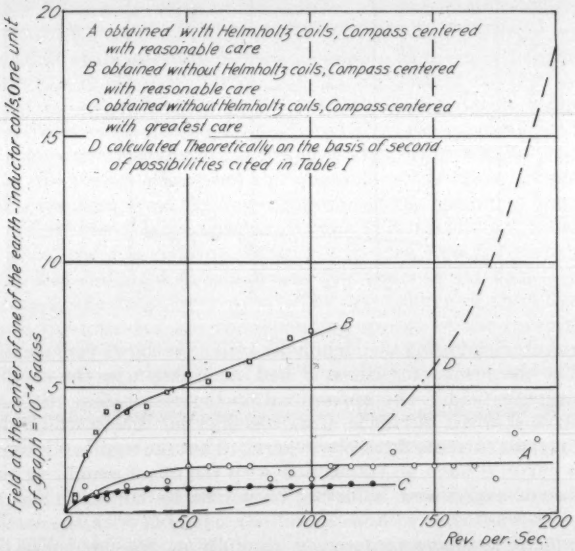


mutator. One end of the coil system was attached to the metal rod A, which rotated in the mercury cup B. The other end was attached to the thin copper cylinder C which rotated

in the mercury cup *D*, which formed an annulus concentric with the cup *B*. Hard rubber formed the mounting for the cups, and a hard rubber tube separated *A* from *D*. The resistance of the two earth-inductor coils was about 600 ohms; and, with the commutator as described, was the same under rotation as at rest.

The use of an alternating current galvanometer in conjunction with an amplifier calls for attention to many details if one is to interpret properly the relation between the input and output. However, in the present instance, all uncertainty in this connection was avoided by the process of calibrating the system under its working conditions by means of the coil *C*, Fig. 1, *B*.

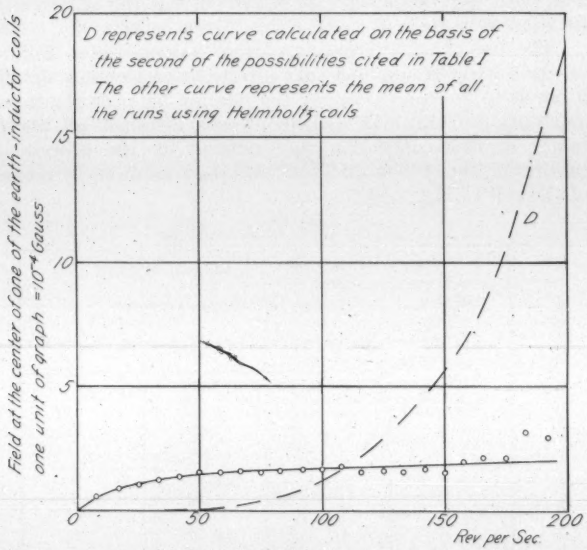
FIG. 5.


A obtained with Helmholtz coils, Compass centered with reasonable care
B obtained without Helmholtz coils, Compass centered with reasonable care
C obtained without Helmholtz coils, Compass centered with greatest care
D calculated Theoretically on the basis of second of possibilities cited in Table I
Field at the center of one of the earth-inductor coils, One unit of graph = 10^{-4} Gauss
Rev. per. Sec.
0
50
100
150
200
5
10
15
20
A
B
C
D

The Results: Figs. 5 and 6 give the results obtained, the ordinates representing the field intensity at the center of one of the earth-inductor coils, and the abscissæ representing the

speed of rotation. Curve *B*, Fig. 5, gives the results obtained in the absence of the Helmholtz coils, and with the earth-inductor coil centered with no special care. Curve *A*, Fig. 5,

FIG. 6.


D represents curve calculated on the basis of the second of the possibilities cited in Table I
The other curve represents the mean of all the runs using Helmholtz coils
Field at the center of one of the earth-inductor coils
one unit of graph = 10⁻⁴ Gauss
Rev per Sec.
D
0
5
10
15
20
50
100
150
200

was obtained using the Helmholtz coils. It shows very clearly that the results for curve *B* had their origin in the earth's magnetic field. The approximation to a saturation effect in curve *B* arises, of course, from the shielding action which the Foucault currents themselves exert. That the results obtained in curve *B* have no connection with the effect sought in the present experiment is further borne out by curve *C*, Fig. 5, which was taken without Helmholtz coils, but with the earth-inductor compass centered as carefully as possible. We do not attach any special significance to the fact that the curve *C* lies lower than curve *A*, as imperfect centering of the earth-inductor compass may, in the presence of the earth's

field, cause deflections either positive or negative, depending upon the nature of the lack of symmetry.

Curve *D*, Fig. 5, represents the results which would have been obtained upon the basis of the second of the possibilities cited in Table 1, viz., that in which the apparent current density is proportional to $Dr^3\omega^4$, the factor of proportionality being adjusted so as to result in 0.5 gauss for the pole of the earth. The curve *A* obtained with Helmholtz coils thus corresponds to a field only about one-ninth of that calculated for a speed of 200 revolutions per second on the basis of the above formula. One is naturally tempted to inquire as to what the residual field represented in curve *A* may correspond to. It disappears when the sphere is removed, and only the shaft and the remainder of the apparatus is in operation. Thus, it owes its origin to the sphere. It was found possible to produce a spurious effect of this kind by placing a finger on the shaft of the earth-inductor compass, so as to cause a slight distortion of the axis. It is clear that very slight relative alterations of the planes of the coils of the compass during a run would serve to give quite large effects unless the magnetic field of the earth were compensated absolutely. It is thought that this effect may arise from the presence of the wind on the coils, resulting from the rapid rotation of the sphere.

Of course, a large number of runs were taken, and a list of all of them is given in Table 2. In each case the earth's field was balanced to about one per cent. of its value by means of the Helmholtz coils. The last column of the table gives the averages, for corresponding speeds, of the residual fields for all of the runs. In view of the fact that the number of runs at high speed was less than that at low speed the number of observations which contribute to the average gets less and less with increase of speed. The average values are plotted in Fig. 6, again with the theoretical curve *D* for comparison.

Experiments designed to measure the possible production of a magnetic field as the result of rotation were performed by P. Lebedew,[4] who used for the purpose a cylinder 3 cms. in radius and 6 cms. long. He rotated this cylinder up to 500 revolutions per second, and tested for a magnetic field by

[4] *Ann. der Physik*, 39, p. 840, 1912.

TABLE 2.

Record of All Runs Using Helmholtz Coils.

The numbers under H, when multiplied by 10^{-4} give the field in gauss at the center of one of the earth-inductor coils.

Date	Mar. 10	Mar. 12	Mar. 13	Mar. 13	Mar. 13	Mar. 19	Mar. 23	Mar. 23	Mar. 23	Mar. 29	Mar. 29	Mar. 30	Apr. 2	Apr. 2	Apr. 2	Apr. 2	Average
Speed r.p.s.	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H
200																	
192					2.92												2.92
183					3.19												3.19
175				2.9	1.33												2.12
166				2.4	1.86												2.13
158				2.1	1.86												1.98
150	2.56	1.63		1.6	1.86												1.53
142	2.26	1.63	1.3	1.3	1.86			1.79									1.69
133	2.42	1.40	1.36	1.6	2.40			1.22	.28								1.53
125	2.21	.47	1.86	1.6	1.86			.94	.47					2.02	2.64	1.90	1.60
116	2.21	.47	1.86	1.6	1.86			.66	.47					1.98	2.55	1.92	1.56
108	2.26	.47	2.9		1.86			1.13	.57		2.31			1.94	2.50	1.94	1.79
100	2.42	.70	2.1	1.6	1.33		2.58	1.03	.57	1.95	2.26	.86	1.51	1.90	2.46	1.92	1.68
92	2.72	.47	2.5	1.3	1.59		2.53	1.03	.57	1.89	2.21	1.12	1.14	1.86	2.46	1.74	1.68
83	2.36	.47	1.8	1.3	1.33		2.48		.47	1.89	2.15	1.72	1.06	1.82	2.38	1.66	1.64
75	2.21	.93		1.85	1.86	.30			.47	1.89	2.10	1.63	.87	1.81	2.36	1.70	1.54
67	1.96	.47	2.9	1.6	1.86	.78	2.48	.85	.47	1.78	2.00	1.63	.99	1.75	2.34	1.66	1.60
58	2.21	.47	2.9	1.6	1.86	1.20	2.53	.56	.47	1.73	1.94	1.59	−.20	1.71	2.27	1.66	1.53
50	2.21	.93	2.7	1.6	1.86	.90	2.48	.56	.47	1.73	1.79	1.55	0	1.61	2.18	1.60	1.51
42	2.26	.23	1.3	1.6	1.26	1.08	2.43	.47	.47	1.62	1.68	1.42	.08	1.50	2.04	1.55	1.35
33	1.06	.47	1.3	1.3	1.59	1.02	2.38		.47	1.51	1.52	1.33	0	1.38	1.91	1.45	1.25
25	.76	.23	1.3	1.1	1.06	.90	1.91	1.13	.47	1.35	1.37	1.25	.24	1.24	1.68	1.43	1.09
17	1.51		1.6	.2	.53	.84	2.10	1.22	.38	1.08	1.05	.95	.51	.97	1.38	1.23	.91
8	.30	.47	1.6	−.2	1.06	.30	1.81	−.28	.38	.60	.58	.47	0	.47	.79	.89	.58
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

means of an astatic magnetometer. His object was to test for a magnetic field which, at the pole of a sphere of radius a, would be proportional to $\omega^3 a^3$. No appreciable magnetic field was observed. His sensitivity was such that on the basis of the second type of law quoted in Table I, he would, for the most favorable case—that of his brass cylinder—have measured about 3 divisions on his magnetometer scale at the highest speed of rotation, 500 revolutions per second, whereas the sensitivity of our apparatus was such that, on the basis of the same law, the expected effect amounted to from 500 mm. to 70 mm., at 200 revolutions per second, depending upon the sensitivity adopted in the particular run concerned. The actual sensitivity of our earth-inductor compass varied in different experiments from 2.6×10^{-5} gauss per division to 0.4×10^{-5} gauss per division, while the sensitivity of the astatic magnetometer of Lebedew was 4×10^{-4} gauss per division, as estimated from the data in his paper. Neither in the experiments of Lebedew, nor in ours, would any measurable effect have been expected on the basis of a law of the first type quoted in Table I, with constant of proportionality adjusted to correspond to the earth's field.

It is worthy of note that, in expecting a correspondence between the form of the law for the conditions pertaining to the earth and those pertaining to a copper sphere, 10 cms. in radius, rotating 200 times per second, we are extrapolating over an enormous range. The situation may be illustrated by supposing that the apparent current density, instead of being proportional to $r^3\omega^4$ were proportional to $r^3\omega^{3.8}$. This very slight difference in the power to which ω is raised would result in an alteration of the field predicted for the small sphere from 43×10^{-4} gauss, to 1.5×10^{-4} gauss at the pole, and the latter field would not be measurable with the apparatus used. On the other hand, this small change in the form of the law would be negligible in its importance as regards comparisons of the magnetic fields of bodies such as the sun and earth, whose angular velocities are not of a totally different order of magnitude.

On account of the necessity for stability, the final experiments here recorded were performed at Swarthmore College instead of at the present temporary site of the Bartol Research

Foundation in Philadelphia. Our sincere thanks are due to Acting President John A. Miller for permission to use a laboratory at the College, also to Professor Winthrop R. Wright and Professor Andrew Simpson for the provision of facilities. Finally, we wish to acknowledge the efficient services of the chief instrument maker of the Bartol Foundation, Mr. Oscar Steiner, who made most of the apparatus and assisted in taking the measurements.

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TOPOGRAPHY FROM THE AIR.*

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WHEN the science of map-making from aërial photographs is considered from the viewpoint of its actual practical applications, it will be found that the problems arising therewith can be classed in a number of distinct groups. Each of these groups is independent from the others to the extent that any solution of the problems in one group may be combined with any solution of the problems of either of the other groups to accomplish the final purpose of constructing a topographic map by means of the data furnished by aerial photography.

In their rational sequence these groups are the following:

1st. The production of aërial photographs.

2d. The determination of the position of the focal plane of each of the photographs with relation to the earth.

3d. The conversion of the views as originally exposed to their equivalents in a horizontal plane of projection.

4th. The assembling of a traverse from the separate views to one aggregate area for the horizontal control.

5th. Construction of the contour lines.

6th. Converting the central or conical projection of the photographic views to orthogonal projection of the map.

7th. Drawing and reproduction of the actual map.

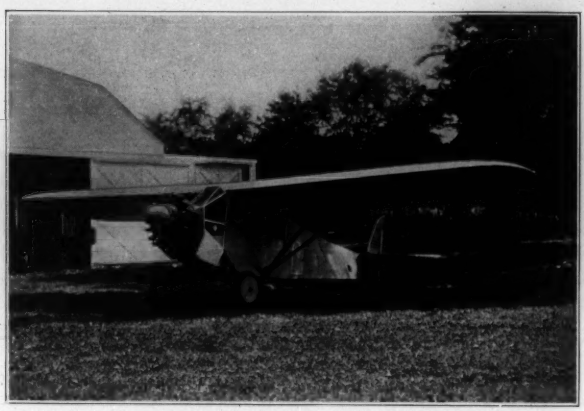
A discussion of all solutions which have been proposed in each of these groups, and many of which are in practical use, would lead far beyond the time limits available and it will therefore be necessary to deal with some selected ones of special interest on account of their adaptability to conditions found in this country.

Taking them up in the order just stated, we at once realize that aërial photographs must necessarily be made from suitable aircraft. The lighter-than-air craft, desirable as it may

* Presented at a Meeting held Thursday, February 23, 1928.

be from the viewpoint of steadiness of motion, is ruled out on account of its cost of operation, which would have made aërial topography too expensive to ever have become a com-

FIG. 1.



The photographic air-ship.

mercially sound industry. There remains thus the aëroplane, which, strange as it may appear, has until quite recently been constructed with an almost complete disregard for the special features required to effectively expose a series of views suitable for mapping purposes. Among the various types of aëroplanes, those suitable from the viewpoint of carrying capacity, operating space, ceiling, flying speed and landing speed, none were quite satisfactory as to the visibility afforded the pilot to steer his machine with the necessary certainty and precision over the lines of flight he is to follow. Whereas in general it is feasible for a well-trained pilot to follow the first of a series of flight directions, attempts to make further flights parallel to the first and at a predetermined distance were always highly uncertain on account of the impossibility of the pilot's seeing the actual course of his ship over the ground, however, recently a new design of aëroplanes has

been perfected, in which this peculiar difficulty has been satisfactorily overcome.

As to the stability of the camera, no matter if the photographic mapping process uses vertical, oblique or horizontal pictures, (for which designations the position of the lens axis is the determinant factor), it is essential that the exact position of the focal plane be known before further use can be made of aërial photographs for map-making.

FIG. 2.

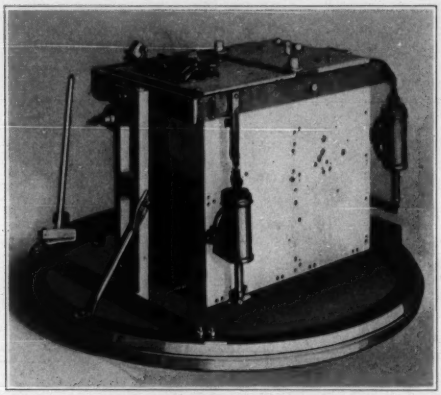
The Brock camera mounted in the airplane.

At first many attempts were made at stabilizing the camera, with relation to the earth, but thus far no really effective solution has been found along this line and hence the question arises, when given an aërial view, how can its plane of projection and the location of its viewpoint in space be reconstructed from the view itself, from the only actual information we have about it, viz: the focal length of the lens which made it and the position of this lens relative to the view. This brings us to the second group of problems:

the determination of the position of the focal planes of a series of views with relation to the earth.

In as much as this group of problems presents probably the most important one among the strictly theoretical questions in consideration, it has received attention from a large number of workers in this line of endeavor, with the result that several solutions have been offered. They may be classed in two main divisions; the first seeks to determine for each

FIG. 3.



The Brock camera on its turn-table.

view its true position by comparison of the view itself with the territory it depicts; that is, a comparison of some of the image points with the actual object points to which they correspond. The general principle underlying this class of solutions is based on what is known as projective relationship. This exists whenever rays proceeding from certain image points through the focal point of the lens all pass through their respective object points, in the reverse direction in which they proceeded from the object points to the image points at the time of exposure.

The actual image plane position can be correctly repro-

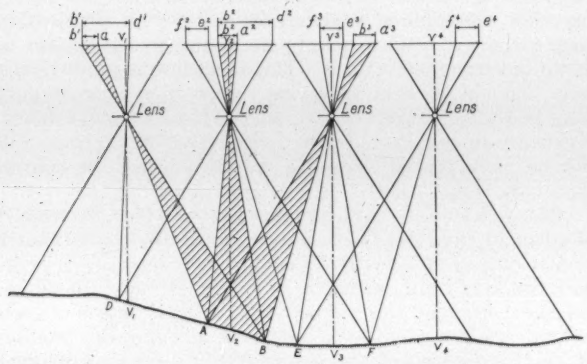
duced in this manner when the drawing containing the points representing the original object points is correct. It is necessary to have such a drawing in convenient scale, as it is manifestly impossible to use the actual object points themselves. Making such a scale drawing involves the measurement of the three space coördinates of each one of the ground points to be located on the drawing. At least four points per view are required to determine perspective relationship with certainty, as three points are frequently insufficient to avoid indeterminate values. This procedure requires thus a large amount of field surveying, unless the required data may be obtained from existing maps of sufficient reliability. Evidently in this instance the resulting air-survey map will not be more precise than the one from which the control data have been taken.

Many European countries possess maps which are deemed of adequate precision to be so used, but in the United States similar data of sufficiently dependable character are in existence in very rare instances only. It became, therefore, expedient to avoid this method of direct comparison of each separate view, and to look for means whereby the position of the focal plane of the several views can be established without the need of such extensive ground survey. The search for a method, which, while theoretically correct would at the same time be free from prohibitive expense for gathering field data, resulted in the Brock process, now being operated by the firm of Brock and Weymouth, Inc. of Philadelphia. Their method belongs to the second class in this group, in accordance with which aërial views are compared pair-wise with each other, instead of one by one with the ground. The stereoscopic relation between adjoining overlapping views forms the criterion whereby the true position of each view is determined. The Brock process uses, therefore, vertical views, that is to say, views made with vertically downward-pointing lenses on horizontal focal planes. Although the exposures are made as nearly as possible with correctly vertical lens axis, that condition is practically never realized and even if it were accomplished, it will only be found to be so by the same tests which indicate how much the deviation from verticality has been. The only difference is that the

answer will be zero instead of the usual deviation of a few degrees or some minutes.

The use of levels to indicate the position of the camera is entirely misleading, since the liquid in the level and the position of the bubble are equally subject to the action of the

FIG. 4.

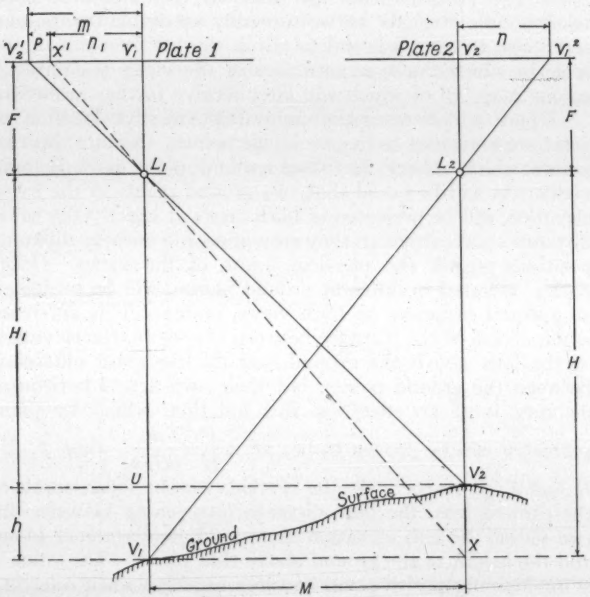


disturbing forces as the camera itself—especially the effect of centrifugal forces by which the direction of the gravitational pull suffers deviation from the vertical, leads to erroneous conclusions. In case a constant centrifugal effect is operative during an exposure period, the camera swings out and the bubble will assume the same relative position as for true horizontality. An actually tilted view would thus be marked as if it were on a horizontal plane. Consequently, such indications are of no dependable practical value.

If, however, the interval between succeeding exposures be so regulated in relation to the angle of view of the lens and of its elevation above the ground, that so much of the image on the first plate of a pair is repeated on the second plate, that the foot point of the vertical through the lens in the first position is still included in the succeeding view (with the necessary corollary that the first image will include the foot-point of the lens perpendicular of the second view) the section of the country between these perpendiculars will occur

in both views so as to create a stereoscopic pair—and it will be shown that it is possible to predetermine the relative position of corresponding stereoscopic points on the two plates, without determining the absolute position of those points on either plate individually. Moreover, to determine such rela-

FIG. 5.



tive positions only the difference in the elevation of the corresponding ground points need be known. The field work is thus reduced to gathering a series of level determinations, without need of any data for the relative location of such points. The consequent reduction in the required field work is of the utmost importance for the commercial aspects of the method.

For the sake of completeness, be it here stated, that besides

the lines of level observations to be made in the field, it is necessary to obtain at convenient intervals, which are always large, a few actually measured distances between two ground points, of known elevation, to serve for the determination of the lens elevation above these base points, and also to determine the resulting scale of the view in that particular plane. Such base measurements are made at ten, fifteen or even twenty mile intervals, between readily accessible points, and serve also to establish and to check the maintenance of the scale in which the aggregate area of the views is combined to one map, all of which will later receive further attention.

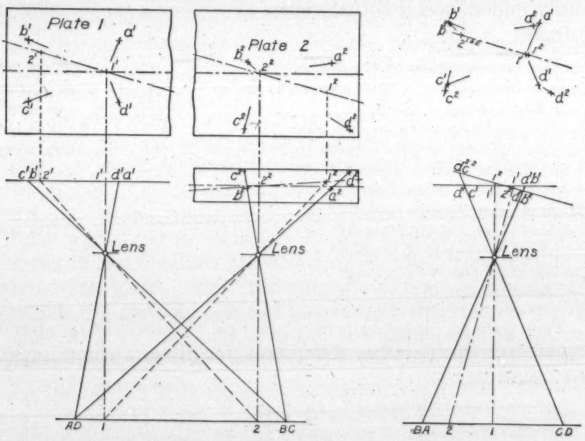
Figure 5 illustrates the compulsory relative location of what are known as conjugate image points; that are the two points which depict the same ground point on adjoining views. It will be noted that two ground points in the same elevation will be projected in both views at exactly the same distance apart, although they may appear in entirely different positions within the physical limits of the views. Other points, situated in different ground planes, will be projected at unequal distances on both views, points. This *difference* is a function of the distance between the views, the elevation of the lens above the ground and the elevation difference between the ground points, but their own actual horizontal distance is of no effect on this function, which by plain geometry can be proven to be: $M \times \frac{F}{H} \times \frac{h}{H \cdot h}$, when F, M, H and h are respectively the symbols for the focal length of the camera lens, the base distance intervening between the two views, the lens elevation above a chosen reference plane and the height of the ground above that plane. The value h is intrinsically positive and becomes negative when referring to points below the reference plane. In each particular case the value $\frac{MF}{H}$ represents the scale of the photograph and can be measured on the view. Using the symbol $\cdots b$ for the image of M, the above equation takes the form $\frac{bh}{H-h}$ or $b \times \frac{h}{H-h}$, from which it will be at once noted that for any given pair of views h is the only variable, and that for stated values of

the fraction $\frac{b \times h}{H - h}$, i.e. for a given amount of parallax difference, the corresponding value of h is determined.

We can thus now stipulate that the meaning of the word "parallax" as applied to vertical aërial views is the difference in distance between two pairs of conjugate image points due to the difference in elevation of the two corresponding ground points. After this we can continue to study the manner in which this phenomenon may be used to determine the position of the image planes of areial views at the instant of their exposure.

In the actual practice of photography from aëroplanes, the image planes are not horizontal, but something on the order of what is shown by Fig. 6 will occur. So as not to

FIG. 6.



unnecessarily complicate the problem, only the right-hand view has been shown as tilted, to illustrate how the distance between image points in a tilted plane become different from what they are on the corresponding horizontal plane, which is shown in dotted lines. Let it be assumed that a pair of

views were exposed as here illustrated, and that the relative elevations of ground points *A*, *H*, *D* and *G* have been ascertained in the field. *H* and *D* are in the same elevation and the distance *HD* has been measured to furnish data for the lens elevation *HO* above the plane *HD*, which is then $\frac{HD}{h^1d^1} \times F$.

Since the amount of tilt is as yet unknown, it is assumed that the distance between the center point *C*1 of the left-hand plate and the conjugate *C*'2 of the center point *C*2 of the right-hand plate is the actual distance between the two view points O_1 and O_2. Having ascertained by measurement in the field what the relative elevations of the ground points *A*, *H*, *D* and *G* are, it can now, with the aid of the parallax equation be determined what, the parallax differences between the corresponding image points must be. It will be most easily understood if actual values be assigned, for instance as follows:

Distance measured between *H* and *D* ..... 1,345 feet.
Distance measured on the plate between *h*1 and *d*1 ..... 1.984½ inches.
Focal length of lens ..... 8½ inches.
Resulting elevation of *O* above the plane *HD* ..... 5,735.96 feet.
Distance measured on plate between $c1_1$ and $c1_2$ ..... 3.222 inches.
Height of point *A* above plane *HD* ..... .44 feet.
Height of point *H* above plane *HD* ..... .0 feet.
Height of point *D* above plane *HD* ..... .0 feet.
Height of point *G* above plane *HD* ..... .21 feet.
Calculated parallax *H*–*A* = .0249″.
Calculated parallax *H*–*G* = .0116″.
Calculated parallax *H*–*D* = 0″.

This pair of plates is now placed on the measuring stereoscope and the parallax differences for these various points are found to be:

$$H\text{–}A = .0269 \text{ error} + .002,$$
$$H\text{–}G = .049.6 \text{ error} - .038,$$
$$H\text{–}D = .008.8 \text{ error} - .008.8,$$

and comparison with the computed values, indicates the errors here shown.

Not only does the presence of these errors indicate that

either one or both of the views are tilted out of horizontality, but their magnitude and sign in conjunction with the location of each of the observed points permit to reach some very valuable conclusions about the magnitude and the direction of the tilt angle in either of the views by the following method of reasoning:

Bearing in mind that the stereoscopic measurements are made on glass positives prepared from the original camera negatives, and that the observer's eyes occupy positions comparable to those held by the lens, so that we are looking down from the viewpoint of the exposure toward the earth, the circumstance that the errors noted in the locations of points *D* and *G* are negative is indicative of the fact that these points are too near the center of plate No. 1, while the small positive error in point *A* is indicative that *a*1 is too far away from that center. We further note that the conjugate point a^2 on plate No. 2 is quite near to the margin of this plate and that consequently the right-hand view has been very nearly level, as otherwise the point *a*2 would have been displaced enough to create a far greater error in the measurement *a*1–*a*2 than was observed.

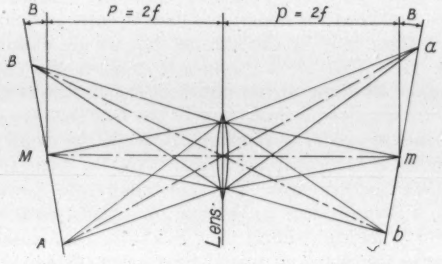
Relative to *g*1 and *g*2 these conditions are just reversed. In this case an error of considerable magnitude exists and since *g*2 is not far from the vertical axis passing through *C*2, the point *g*2 could only have suffered slight displacement as a result of the tilting of the right-hand plate, and it is thus fully warranted to attribute the discrepancy in the measurement of the distance *g*1 *g*2 mainly to the tilt existing in the left-hand plate. Such tilt must have been in the direction which brought the point *g*1 too near to the apex of the light cone, that is the portion of the left-hand plate on which *g*1 occurs was too near the lens, and the plate was tilted *in* on its right-hand side.

An examination of the error in the distance *d*1 and *d*2, which is smaller but of the same sign as the error noted in *g*1, tends to confirm the conclusion just made. Furthermore, the locations of the points *g*1 and *d*1, relative to the vertical axis passing through *c*1, are quite compatible with the relative magnitude of the errors observed in these points. This leads to the conclusion that the errors are probably due to inward

tilt of plate one around the line passing through $c1$ at right angles to the common center line $C1$ $C2$ without additional tilt around the latter line.

The negatives of plates 1 and 2 are now inserted in the correcting projector and plate No. 1 is adjusted so as to bring the estimated tilt axis in coincidence with the tilt axis of the

FIG. 7.



projector, and its focal plane inclined to the angle corresponding to the estimated tilt angle, leaving for the time being the focal planes of the projector containing the right-hand negative at right angles to their lens axis.

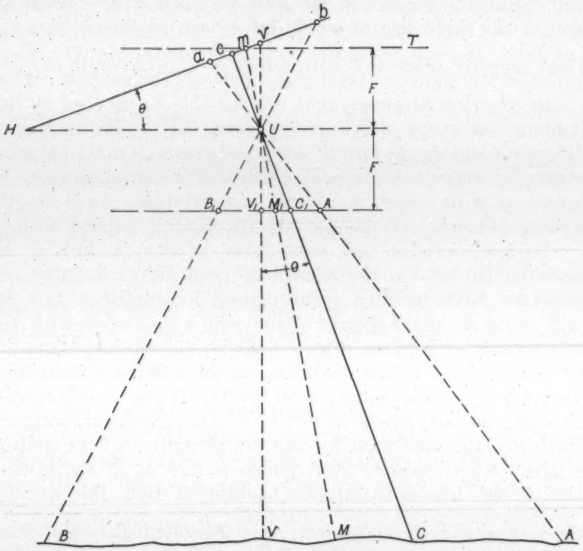
CORRECTING PROJECTOR.

The angular adjustment given to the left-hand negative has been automatically accompanied by the corresponding angular adjustment of the image receiving screen in the positive focal plane, and further by such decentration of the negative and of the screen as is necessary for the former to insure freedom of distortion and for the latter to indicate the vertical point in the image projected on it. This permits to determine the exact effect of the changes resulting from the tilted projection. With the aid of Fig. 4, the necessity for these adjustments will be made clear.

The inclined plane $Ha.C.$ contains the tilted negative $a.C.$ produced with a lens f of the focal length Uc. The horizontal plane HU through the optical center of the lens is the horizon plane of the tilted image, and the point H is the vanishing point for parallel horizontal lines. The dotted hori-

zontal plane B_1A, at the distance f below the viewpoint U, contains the correct positive image and corresponds not only to the tilted image plane $a.b$, but also to the ideal negative

FIG. 8.



plane mT at a distance UV equal to f above the viewpoint U. This ideal image plane intersects the tilted image plane in a line passing through the point m, which line and point are thus common to both planes. Consequently, various image points found in this line have suffered no mutual displacements by being projected on the tilted plane. The point m lies of necessity in the bisector of the tilt angle θ, from which it will be noted that the triangle HUm is isosceles, since the angles HmU and HUm are equal by construction, viz: $\frac{180° - \theta}{2}$ or $90° - \frac{\theta}{2}$. If, then, the broken line CUM_1 be replaced by a straight optical axis in the direction M_1Um

and a lens having a focal length $\frac{UM_1}{2}$ be substituted in U for the actual camera lens, it will be clear that with suitable illumination of the negative in the tilted plane ab this lens will create the correct image AB_1, its horizontal equivalent, and of the same size as would have been produced from the ideal negative plane mT with a lens of a focal length of $\frac{f}{2}$.

In practice, however, it is not feasible to proceed in this manner, as every different negative tilt would require a different focal length for the corrected reproduction. Mathematically correct horizontal equivalents can, however, be produced with lenses of almost any arbitrary focal length, so long as a few very simple conditions are complied with.

To horizontalize any view it is, of course, first of all essential to remove the vanishing point H to infinity. A projector with inclined focal planes accomplishes this for that point in the negative plane which coincides with the intersection of a plane drawn through the optical center of the lens parallel to the directions of the positive focal plane. No other point will comply with this condition, as will readily be seen from an inspection of Fig. 7, which diagrammatically illustrates the condition for the maintenance of conjugation between tilted planes. The points m and M in the optical axis of the lens are conjugate points so that the distance $Ob = (n + 1)F$, and $Oa = \frac{(n + 1)}{n} F$, when F is the focal length of the projector lens and n the ratio of magnification in the lens axis. If, now, these points m and M are considered to be pivot points for the planes $a.C.$ and AB, which have a common intersection T in a plane UT normal to the lens axis in the optical center U, the two inclined planes will be conjugate over the entire range of the field of thé lens, without any demand on its quality of depth of focus, as may be readily proved by applying the law of conjugate foci to these planes. The same condition is more conveniently expressed by the equation: $tg\,\beta = n.tg.\,\alpha$, in which n is the relation $\frac{UM}{Um}$ that is the ratio of reproduction in the lens axis. If, now,

the image field in the plane *ab* be thought sufficiently extended to include the point *H* produced by a plane *UH* parallel to *AB*., it will be evident that the image of the point *H* will be rejected to infinity in the plane *AB*. It can further be shown that a perpendicular to the line *Mm* through the point *H* cuts the distance *Um* in two parts—*Lm* and *UL*, of which *OL* represents the focal length of the lens *F* placed in *O*. By construction the angle *UHL* is angle β and hence $F = HU \sin \beta$ or $HU = \frac{F}{\sin B}$.

It will also be noted from Fig. 8 that in the tilted original, the distance *HU* of the vanishing point to the lens is equal to $\frac{f}{\sin \theta}$ and since *HU* in the negative must equal *HU* in the projector, we find $\frac{F}{\sin \beta}$ must equal $\frac{f}{\sin \theta}$ or $\sin \beta = \frac{F}{f} \sin \theta$. Since for any given camera and projector the values *f* and *F* are constant there exists thus the simple relation $\sin \beta = R \sin \theta$ when *R* equals $\frac{F}{f}$.

We have already found that the distances *HU* and *Hm* in the negative are equal, and consequently we will know that the point *H* of the tilted negative coincides with the point *H* of the projector, when the point *m* is placed in coincidence with the intersection of the arc drawn with the radius *HU* and the negative plane *a.b*. The image of the point *m* will then be projected on the plane *AB* in the point *M* and the relative scale of its reproduction will be the same as if *m* and *M* had been projected on parallel planes at right angles to the lens axis through each of these points. Since the scale in the *m* line of the tilted negative is, as already stated the same as the scale in the *m* line of the ideal negative, this will also be the true scale relation between the corrected projected image and the ideal original. However, this scale is as yet unknown, and since we must arrive at recognition of the corrected image by comparison of parallax values, it is essential for our purpose that the scale in the *m* line be known. This can most readily be accomplished by so arranging the angles α and β that the *m* point will coincide with the lens

axis. A further inspection of the Fig. No. 7 shows that the arc already mentioned intersects the lens axis in a point distant $2F$ from the lens U. The law of conjugation requires, then, that also the point M must be at a distance $2F$ from U so that $\frac{(n-1)}{n} F$ equals $(n+1)F$. This equality is only obtainable for the value $n = 1$ and consequently in that case $tg\beta = tg\alpha$ and both $\sin\beta$ and $\sin\alpha$ are then necessarily $= R \sin\theta$. We then arrive at the positions illustrated in the Fig. 7, in which the points m and A coincide and consequently also M and B and which yields a reproduction which constitutes the equal sized horizontal equivalent of the ideal image plane mT.

Coincidence of the point m with the lens axis requires that the optical center of the negative C_1 be decentered toward T by the amount: $f \tan \frac{\theta}{2}$. When this point is set off on the line, Hm and a line drawn through U and C_1 is produced until it meets the plane MT, the intersection locates the center C on the corrected image. If, now, the screen upon which this image is received be decentered from its normal position also through a distance $f \tan \theta_2$, the screen point which originally was in coincidence with the lens axis will then indicate the true location of the v point on the corrected projected image. This allows to ascertain the distance $V_1C_2^1$ on this image, which for the given adjustment represents the true picture base intervening between the two projected views. With this information the lens elevation E can be recomputed and consequently new values for the parallax differences between the points A, H and G can be established and measurements of these parallaxes on the two projected images compared thereto.

Assuming that the focal length of the projector lenses is $9\frac{1}{2}$ inches, the angle β would in the present example have been adjusted so that $\sin\beta - \frac{9.5}{8.5} \times \sin 1^\circ\, 30' - 1.1177 \times .026177 = .029258$ or $\beta - 1^\circ\, 40', 35'\, 69''$ and inversely when an adjustment of β to this angle is found to produce an image indicating the correct parallax differences, the tilt angle is derived by the equation $\sin\theta = \frac{\sin\beta}{R}$.

If, upon the first tentative adjustment of the tilt angle β the resulting image does not yet show full compliance with the conditions of horizontality, the changes which have occurred will readily suggest to an experienced operator which additional adjustments will lead to the desired result.

The operations just described could not be carried out successfully without means in the correcting projectors to continuously decenter the view in accordance with the value $ftg\frac{\theta}{2}$ for any angle, which according to the relation $\frac{\sin}{R}\sin = O$, corresponds to any tentative angular adjustment of screen and negative. Intricate as this relation may at first glance appear, it is readily complied with by the simple automatic decentration mechanism applied to the correctors.

Having reached adjustments, which produce images that indicate horizontality of the image plane by proper agreement of the parallax differences, the screens are removed and sensitive plates inserted to receive the corresponding exposures, which are then used for the further step required in the actual production of a map. These plates are now ready for the several operations in the fourth group of activities, which deals with the assembling of the separate views to the aggregate of the surveyed area. Much of the work to be done under this heading has progressed alongside of the labors involved in carrying out the process of correcting.

This fourth group comprises everything pertaining to the horizontal control of the map, and insures the proper relative location of the views contained on the various plates and the maintenance of the true uniform scale of the map to be derived from the views.

In the over-great majority of instances, positive prints on glass of the uncorrected camera negatives may be used for this purpose, as in general the tilt in any plate is so small that the resulting directions from certain selected points to the optical center of the plate are practically unaffected. The result of tilt is to displace image points radially in or out, thereby disturbing their mutual distances, but the directions connecting each point with the assumed center of projection are hardly changed. Furthermore, if inadvertently a plate may have been more severely tilted, the resulting

measurable errors in direction become at once manifest and the work of assembling the traverse or plot is halted until the corrected plate can replace the badly tilted one. A further feature, which makes the use of uncorrected originals quite practical for this particular purpose, is found therein that the scale of successive views is absolutely immaterial, as only directions and no actual distances come into play in this branch of the work and the scale of the map is entirely independent from the scale of the negative. In practice the plates are exposed in approximately one-half the scale determined upon for the final map-drawing. This rests, however, on considerations of expediency and economy, as in that manner an enlargement of the original view to two diameters furnishes a plate containing approximately four times the area to the same scale which would result if the photographic flight had taken place at an elevation corresponding to the map scale. Only one-quarter of the number of views is required to cover a given territory and the increased distance between the required control points has been doubled, thereby not only increasing precision but reducing the cost of field operations. Incidentally, the increased point displacements on the enlarged views due to tilt in the original exposure become sufficiently large that deviations from horizontality as small as 0° 2′ may readily be recognized and corrected.

The enlarged views are sorted in groups of three to roughly blue pencil on the first and the last of each such group the extent of their overlap. Plate one and two of the group are then placed on the measuring stereoscope and a series of suitably located image points within the blue pencilled area are stereoscopically picked out and marked on each of the plates under examination. These points are thus found also on the third plate of this group, so that by stereoscopically examining the second and the third plate, the corresponding conjugate points on the third plate in its blue pencilled region are marked with the aid of the already marked points on the middle plate of the group. Also, wherever they appear, the conjugate image points of the ends of a base-line measured on the ground are similarly marked.

Having in this manner selected a sufficient number of points that occur on three succeeding plates, besides a few

which occur only on two plates, their angular directions relative to the lines uniting the succeeding view-points are transferred to the plot drawing. This drawing contains thus, in pencil, a series of radially extending lines, each one indicating the direction from one of the picked points to the center of the plate—also the direction between this center and the centers of the adjoining plates. These directions are, in fact, the azimuth angles under which the various object points corresponding to the several image points would be observed from a ground point coinciding with the optical center point of the view—and are obviously not affected by the relative altitudes of any of these points. If, then, two adjoining plates are superimposed so that the lines uniting the center points of each view are in actual coincidence, the lines radiating from the center of plate No. 1 will intersect the lines radiating from the conjugate image points to the center of No. 2 and in their intersection determine the correct location in orthogonal projection. Sliding one plate over the other parallel to the direction of the common center lines, the various intersection distances will increase or decrease in direct proportion to the increase or decrease of the distance between the points C_1 and C_2, which permits to regulate the scale at will to any predetermined value.

Referring back to the length of the measured base HD, which was found to be 1345 feet, and adopting a map scale of 600 feet to the inch, the distance HD on the map should be $\frac{1345}{600}$ inch $= 2{,}240''$. The plates should then be shifted until the intersection of the rays to the points H and D are exactly that distance apart. In that adjustment the exact intersections of the H and D lines are laid out on the map-sheet, and the points C_1 and C_2 are equally marked. This operation insures the plotted traverse to be in the desired scale. The draftsman now continues to transfer the direction lines toward the various control points, first from one and then from the other of the plates, and when all directions have in that wise been transferred to the plot, draws lines through the points so marked in the direction toward the corresponding centers from which they were marked, thus establishing in the intersection of each pair of direction lines

the true orthogonal location of the corresponding ground point in the correct map-scale.

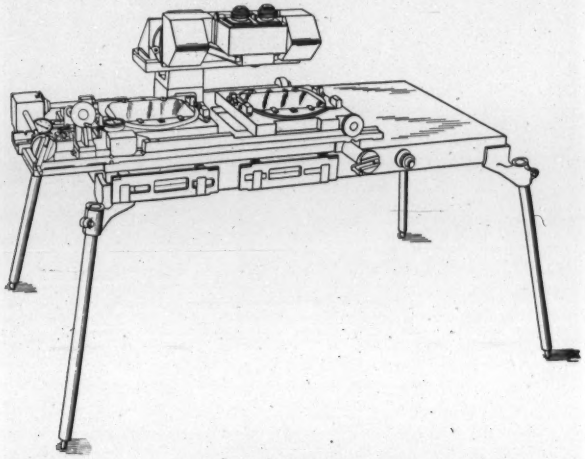
The second plate contains as well the direction line from C_2 to C_3 as from C_2 to C_1—so that when all lines have been drawn, it is now possible to place the third plate so that its direction C_3 to C_2 is in coincidence with the already drawn line C_2C_3. The third plate is then adjusted until the direction marks thereon, to points in the blue-pencilled region, exactly pass through the locations of these points already obtained by the intersections of the first two plates laid down on the plot. It will be readily seen that in this wise a control plot can be built up to extend throughout the area covered by the photographic views. Especially when such areas are so wide that they are built up from a series of parallel strips, the precision obtainable by this procedure becomes very high by the fact that a number of strips may be built up together, giving frequently rise to 5 and 6 intersecting directions to the same point, all of which exactly superimpose themselves. Thereby, the certainty of maintaining correct azimuth directions, as well as maintaining the scale, is greatly enhanced and in practice it is found that on reaching the plotted position of a second control-base, the end points measure with extreme precision the proper distance apart. Such a check covers all of the intermediate point locations, as in the nature of this work errors are not likely to be compensating.

Before taking up the description of the next steps, it is still to be noted that the construction of the traverse, keeping pace in this manner with the correcting of the several pairs of views, permits also to check the flight elevation at which succeeding views have been exposed. If, among the points plotted, be included the several points of which the difference in ground elevation have been measured in the field, the distance between such points as they should appear on any plate for a given scale can at once be determined, and if necessary, the plate in question can then be enlarged or reduced to actually bring such points to the calculated distance.

We now have reached the interesting part, which deals with the production of the contour lines. Pairs of plates have been corrected so that each plate is the equivalent of a

horizontal view in its own viewpoint. The point v previously mentioned and the elevation of each viewpoint of the pair have been ascertained and equalized so that all data are on

FIG. 9.



Comparison stereoscope.

hand to determine for each pair with great precision what the parallax differences must be for points situated at the intervals in which the contours are to appear on the final map. Considering again the pair of plates that have been corrected, and in which the base points H and D are known to be in the elevation of 35 feet above the datum plane of the map, it follows that the lens was $5734 + 35$ feet $= 5769$ feet above the datum, and consequently the parallax for H and D must be $2 \times \frac{1.845 \times 35}{5769 - 35} = 22.4/1000''$ above zero. If the contour interval is to be 10 feet, the first contour line will show a parallax of $2 \times \frac{1.845 \times 10}{5769 - 10} = 6.4/1000$. Assuming the maximum elevation occurring in this pair to be 146 feet, all contour parallaxes from 10 to 140—by steps of 10 feet—are calculated

Fig. 10.

A contoured photographic plate.

and noted in a box provided for that purpose on the plate envelope.

Having properly aligned the plates, the operator brings the image points h^1 and h'' each in exact coincidence with the vertical cross hair of the ocular through which it is seen, and with such separation between the two views sets his micrometer dial to read 22.4. He thereupon reduces the distance between the views until the micrometer reads 6.4 and then begins to look for some place in the view where, with this setting, the cross hairs appear to touch the ground. He has now found a point in the 10 foot contour line, and besides, the physiological effect of the cross hair image suggests to him the location of a horizontal plane cutting the plastic stereoscopic view in the 10 foot plane. He draws the clearly visible contour as far as the view will permit, then shifts the two plates together to bring the end of the line he has just drawn to the cross hairs, checks this line by noting that they have kept in contact with the ground all along the line so drawn, and then continues the line for the next visible stretch. Having traversed in this manner the whole image, and having seen its general appearance, he knows where other places in the view present the same contour, and then continues to draw all he finds within the plate area in this elevation. Should he find the contour to be closed within this area, he proceeds to readjust the plate distance to conformity with the next contour line, which of necessity will also close in upon itself within the area of the one first drawn, and continues until the highest contour is drawn. He then proceeds to draw the 20 foot contour line adjacent to the other 10 foot lines already drawn, and so proceeds to draw one by one all the contours appearing within the stereoscopic area of the views.

With a little training an operator gifted with fair stereoscopic perception can in this way draw contour lines with ease and precision following the minutest indentations, every suggestion of drainage and any and all deflections as they occur in nature, be they high up in the mountains or deep down in the valleys, without moving from his stool behind the instrument, enjoying meanwhile the fascination of the richest detailed bird's-eye view. The work is much facilitated

by the fact that the height dimensions are invariably to a much larger apparent scale than the horizontal dimensions, thereby accentuating elevation differences in the view. This

FIG. 11.

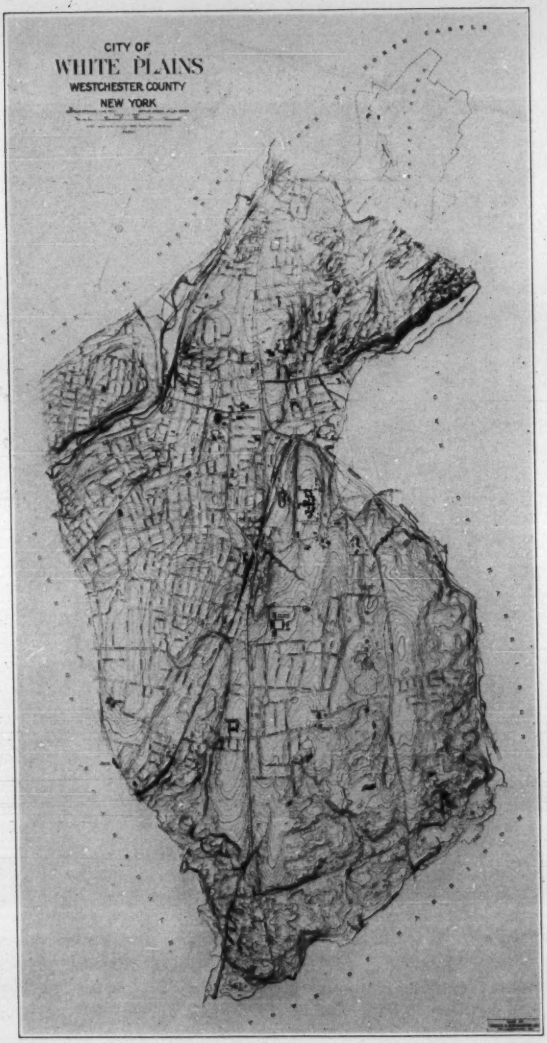
Map of Vecchiano and surroundings, Italy.

phenomenon is due to the favorable ratio which exists between the lens-elevation and the picture distance in comparison with the horizontal scale between the picture base and the pupillary base. It differs, therefore, slightly for different operators, but on an average the scale ratio is about as 1 to

FIG. 12.

Aerial photograph showing the rich detail with which all features of land and culture are reproduced.

FIG. 13.



2.8 in favor of the vertical scale. As this effect is purely psychological, the parallax measurements are in no wise affected by it; it just assists the operator in doing this part of the work. It is evident that contour lines so obtained will automatically assume the upwardly increasing scale of the conical perspective of the view, and are thus not as yet ready for transfer to the plot.

The sixth group of method steps concerns itself with the conversion of the conical or central projection of the views to the orthogonal projection of a map.

This difference is perhaps most readily grasped by comparing photographs and maps respectively with the architect's perspectives and his working drawings. The first show things as they appear, the latter as they actually are. The difference is brought about by removing the center of projection to infinity and therewith removing all perspective from the original views. Up to this point, the presence of perspective was the very thing which permitted to carry out the necessary successive steps, but we have arrived at the proper time for eliminating it, in order to convert the drawings to true uniformly scaled sections of the map under construction. It should be kept in mind that, strictly speaking, the previously described construction of the map-traverse, the actual framework upon which the map is gradually drawn and completed, is the first of the operations for the elimination of perspective, and it appears that at this moment a comparison of the method described with those in Europe will be best understood and appreciated.

Independent from any method whatsoever to determine the actual plane of projection, European constructors have all designed some pantographic machinery, which is used to directly convert the uncorrected view to the corresponding map drawing. They deserve high praise for the ingenuity and perfection of workmanship developed in this respect, but their practice suffers from the circumstance that they can never add more than one view at a time to those previously located, as the very result of their practice of building the pantograph systems in unison with the stereoscopes and operating them by causing the stereoscopic views to pass underneath the comparator marks of the stereoscopes.

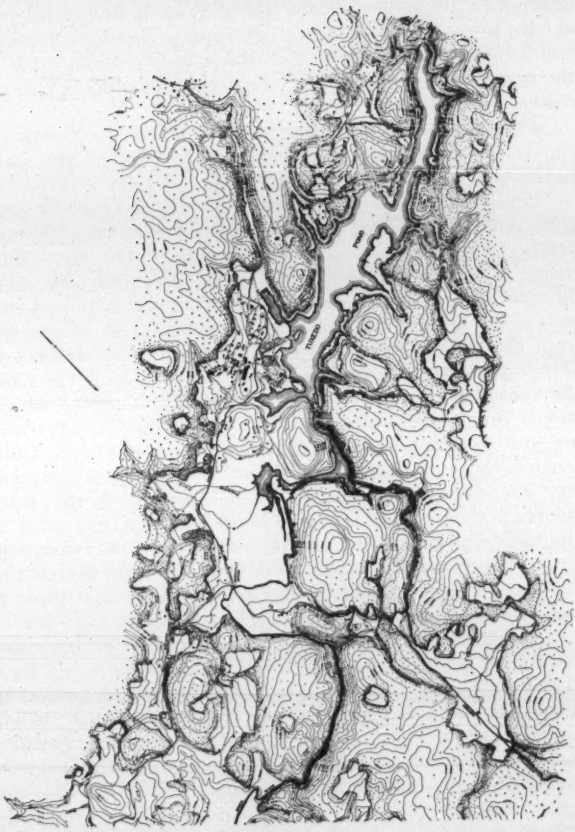


FIG. 14.

Early example of aerial photography.

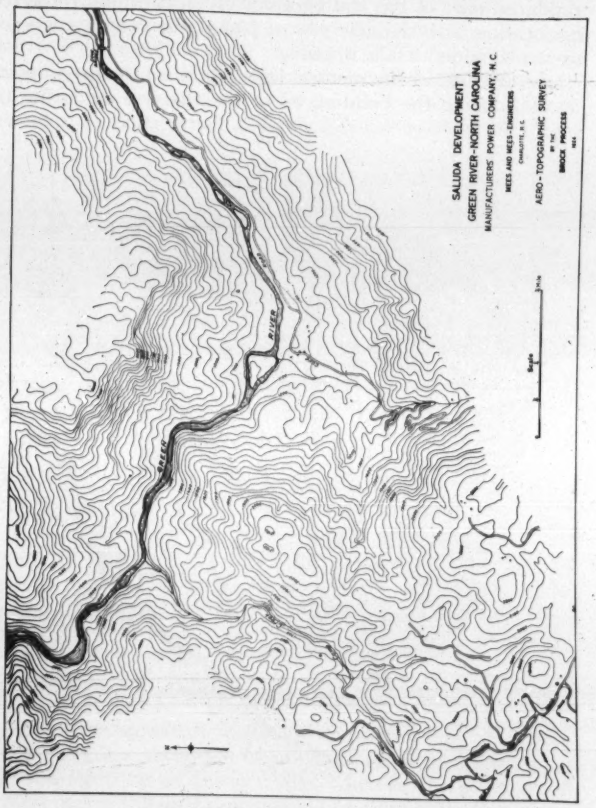
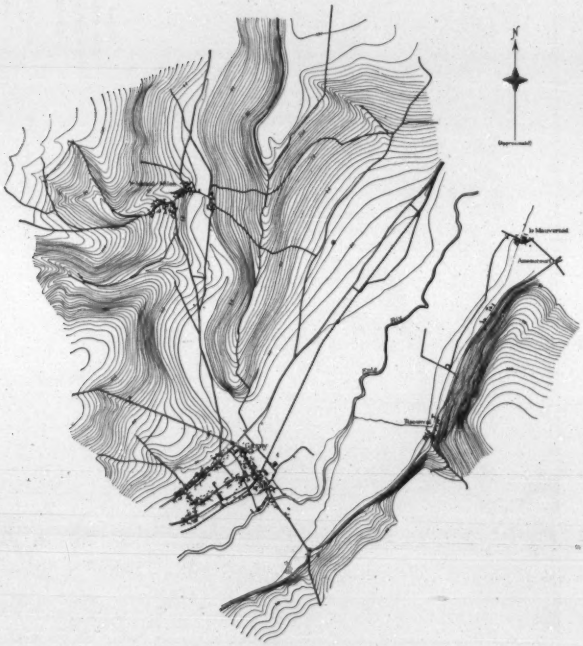

SALUDA DEVELOPMENT
GREEN RIVER-NORTH CAROLINA
MANUFACTURERS' POWER COMPANY, N.C.
MEES AND WEED - ENGINEERS
CHARLOTTE, N.C.
AERO-TOPOGRAPHIC SURVEY
BROCK PROCESS

FIG. 15.

In adjusting a pair of plates to their stereoscopes, they arrive at certain settings and must then adjust the previously produced part of the pantographically drawn map to perfect orientation with the new pair of plates, the contents whereof are to be added to the drawing.

In the case of the method here described to the contrary, as well during the building up of the traverse as during the

FIG. 16.

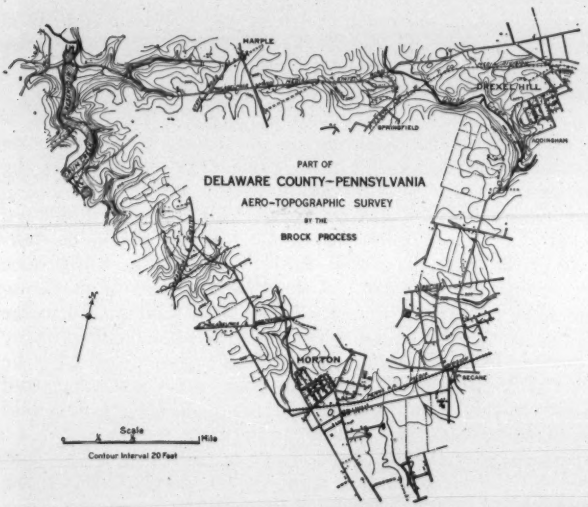


Map of the surroundings of Gasny, France.

transfer thereto of the various map-sections, there are provisionally assembled as many as ten or more sections simultaneously, and thereby a precision in orientation is secured,

unattainable by the European method of procedure.—The ability to produce continuous differential scale equalization is emphasized, whereas in the described method this equalization occurs only zone fashion. Here is, however, a truly and exclusively theoretical point of no practical value, as the very valuable advantage of the high precision in orientation resulting from the American way of proceeding more than offsets the infinitesimally small difference between zonal and con-

FIG. 17.



The first actual areal test-map.

tinuous scale equalization. It furthermore offers the possibility of letting several draftsmen simultaneously work on adjoining sections of the traverse, whereas the pantograph method makes the final map combining essentially a one-man job. This latter point, though being exclusively a commercial consideration, is nevertheless, or rather therefore, a notable advantage of the American method of working.

In this review of the two methods I have assumed that

pantographs operate with 100 per cent. precision. Granted that this may be so as long as they are new and all adjustments have been made with adequate precision, mechanism as intricate and delicate as it must be to avoid mass-reactions, which interfere with free movement, will not for long remain in such perfect mechanical condition that the accumulated back-lash in the screws, gears, sliding members and pivots does not aggregate to more serious displacements of the pencil point than the possible gain of differential scale variation could possibly secure.

Having said this much in defense of the American method of scale equalisation, its actual achievement is best shown by the profiles here shown which represent a check between a map and the ground, for contour intervals of one foot.

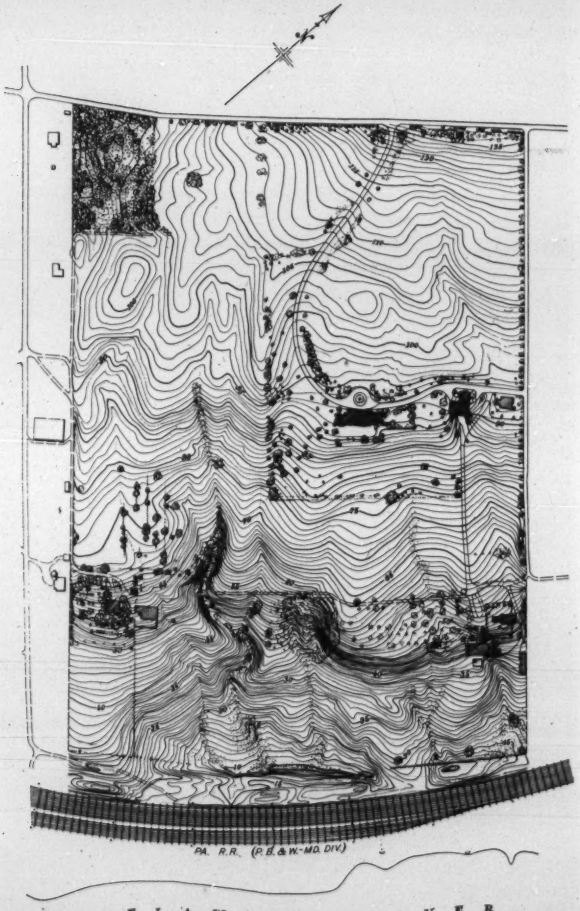
The contour drawing, containing besides the contours as many of the cultural features as are desired to be shown on the final map, is inserted in the camera-end of the equalising projector, which is provided with an automatic focusing mechanism of adequate range on either side of equal-sized reproduction. It projects its image—always on the top surface of the plate glass disk, sunk flush with the top surface of the surrounding table. A sheet of tracing cloth or tracing paper laid flat on this plate glass sheet renders the image visible to the draftsman. The sheet so used has previously been laid over the traverse plot in the region covered by the inserted view so that the true location of the several plotted points of this region are on this tracing when it is first laid on the glass top of the transfer projector.

The size of the projected image is then adjusted so that those of the corresponding points which are included in the inserted contour drawing and happen to lie in the same elevation exactly coincide with the location of the plotted points marked on the tracing. This assures the correct adjustment of the tracing over the projected image, and the magnification is then re-adjusted to the already computed value, which will bring the lowest contour line to the datum plane scale of the map. This contour and the other desired features are thereupon traced, care being taken to continue the roads, rivers and other features not further than half-way to the next adjoining contour line. When this is finished

FIG. 18.

Mosaic showing under water areas.

FIG. 19.



Map showing 1 ft. contours which are checked by the profiles shown on one of the other illustrations.

the image size is re-adjusted to bring the next contour line to the standard scale and the work repeated as before, until the highest contour has in this manner been reduced to its proper scale. The resulting tracing is then a true uniform scale map of the section contained on the contour drawing. The vertical position of this contour sheet in the camera, together with a ventilated lamphouse through which a current of fresh air is forced, allows to let the original remain for the required time without heating it up and destroying its original dimensions within measurable limits. As soon as in this manner a sufficient number of contour sheets has been corrected to drawings of uniform map scale, they are assembled over the original traverse drawing by again registering each one by means of the originally copied plotted points, to examine the exactness of the joints produced by the several sectional sheets among themselves. This never fails to be of remarkable precision, and thereby constitutes a further check on the precision of every preceding step of the process. With suitable weights the several sheets so combined are anchored in position and all but the undermost one are turned back. A sheet of carbon paper of suitable color is laid between this sheet and the traverse drawing, red being used generally when contours are transferred by following the lines of the sealed drawing with a fine rounded steel point. This brings the contours in red on the traverse drawing. A sheet of blue carbon paper is then used to similarly transfer the streams and lakes, and finally with the aid of black carbon paper the various cultural features are transferred. Each sectional drawing being similarly dealt with, the final result is a complete drawing of the map on the traverse plan. As this plan was also derived from the features of the aerial photographs, we possess thus now a truly aero-topographical map of the territory in survey.

As engineering maps are usually desired on tracing cloth, and in equal-sized sheets of standard dimensions adopted by the firms for which they have been produced, sheets of the desired size are then traced from this total traverse and the necessary lettering titles, scales and border lines are added to finally produce the finished product ready to be delivered to the client. It is my pleasure to show you still some slides

of some of the finished work produced by the Brock process, one of which, though small, is truly remarkable in that it shows a tract of ground of approximately 88 acres mapped to an original scale of 200 feet per inch with contour intervals of only one foot. I am certain that thus far no other aerial method has succeeded in accomplishing equal results.

Illustrating the wide range of possible adaptation, figure 21 shows a map of 75 square miles to the scale of 1 inch per 1,000 feet in original construction and reduced to $\frac{1}{48000}$ to fit in with the corresponding sheets of the U. S. G. S. maps.

ON THE DETERMINATION OF STRESS AND STRAIN DURING IMPACT INDENTATION.*

BY

JUNZO ÔKUBO and MASATAKE HARA.

1. INTRODUCTION.

THE present investigation is an experimental work which was undertaken to ascertain the phenomena relating to the impact of two bodies. A short resumé of the previous investigations[1] will serve as an introduction to this paper.

If two spheres, of the same material and of equal mass, collide with equal velocity, (1) the coefficient of restitution rapidly diminishes as the velocity just before the impact increases, and beyond a certain value of this velocity, it diminishes according to a linear law. (2) The time of contact between the impinging spheres also diminishes as the velocity of the impact increases. (3) The volume of the permanent indentation increases linearly in proportion to the loss of the kinetic energy of the impinging sphere. These results were obtained from the measurement of the velocities of the impinging spheres just before and after the impact, the time of contact and the permanent indentation on the specimen.

Notwithstanding that a complete study of the phenomena of impact is very important not only from purely scientific point of view, but also from the technical application, the experiments hitherto reported have been limited to the measurement of the quantities above mentioned and it is very much to be regretted that few attempts have hitherto been made to investigate the phenomena occurring during the contact of the two bodies. To obtain a precise physical conception of the nature of impact, especially the process by

* From the Laboratory of Physics, Tohoku Imperial University, Sendai, Japan.

[1] J. H. Vincent, *Proc. Camb. Phil. Soc.*, 10 (1898), p. 332; C. V. Raman, *Phys. Rev.*, 12 (1918), p. 442. E. W. Tshudi, *Phys. Rev.*, 18 (1921), p. 423; M. G. Moreau, *Journ. d. Phys. et Radium*, 11 (1921), p. 329; J. Ôkubo, *Sci. Rep., Tohoku Imp. Univ.*, Ser. 1, il (1922), p. 454.

FIG. 20.

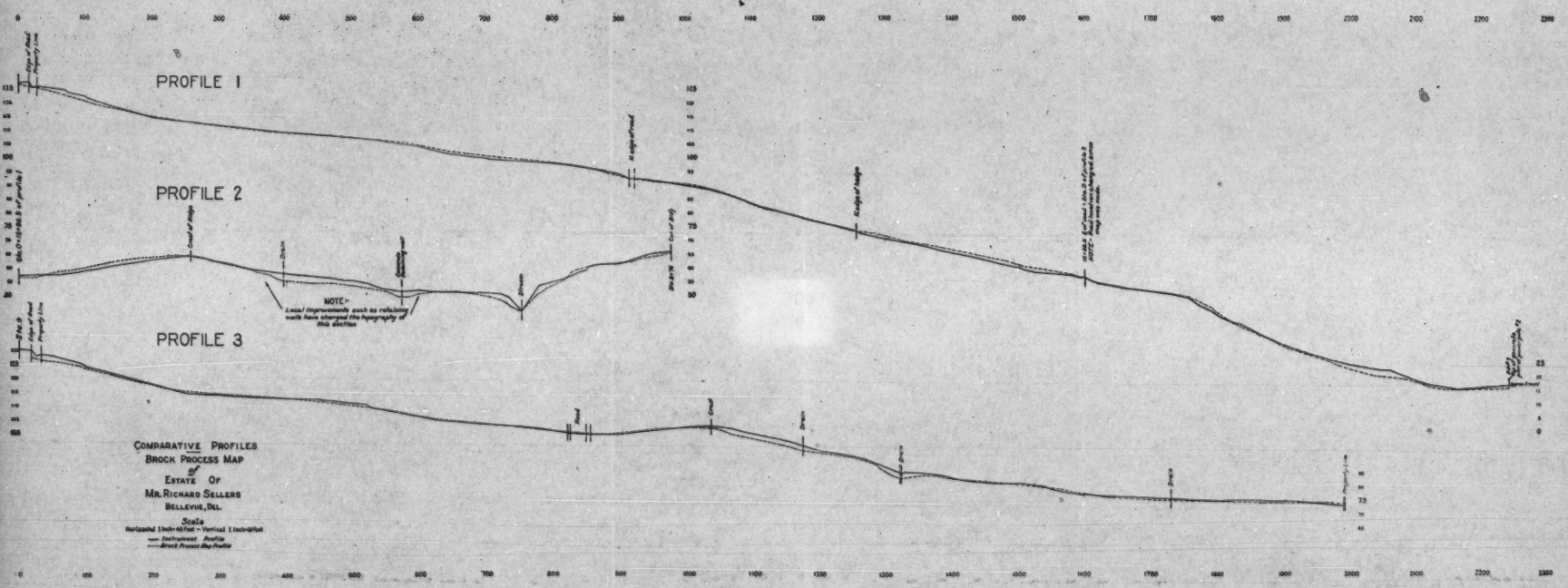
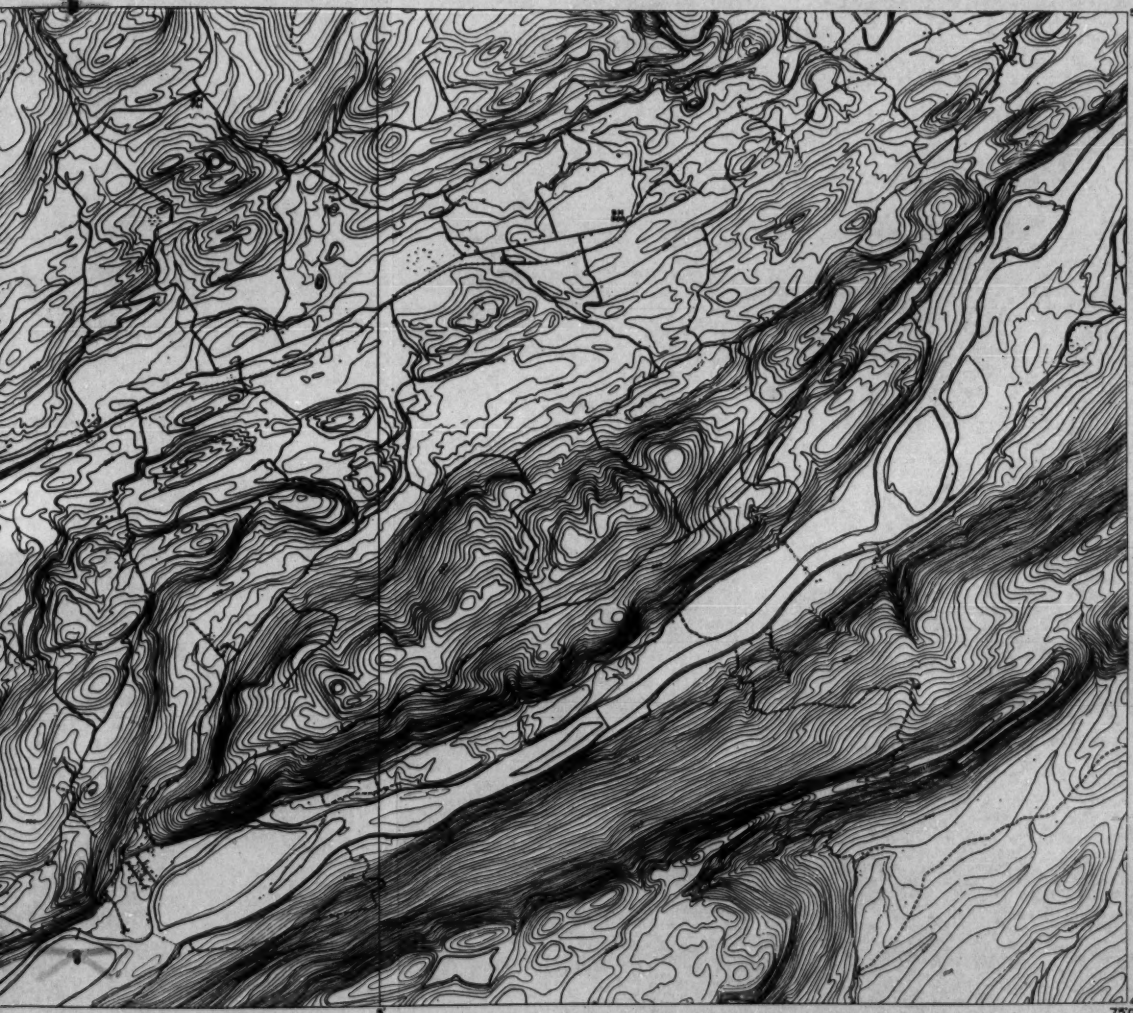

PROFILE 1
PROFILE 2
PROFILE 3
NOTE:-
Local improvements such as refuse
wells have changed the topography of
this section
COMPARATIVE PROFILES
BROCK PROCESS MAP
of
ESTATE OF
MR. RICHARD SELLERS
BELLEVUE, DEL.
Scale
Horizontal 1 Inch=60 Feet — Vertical 1 Inch=10 Feet
Instrument Profile
Brock Process Map Profile

FIG. 21.


5'
5'
4'00"
75'00"

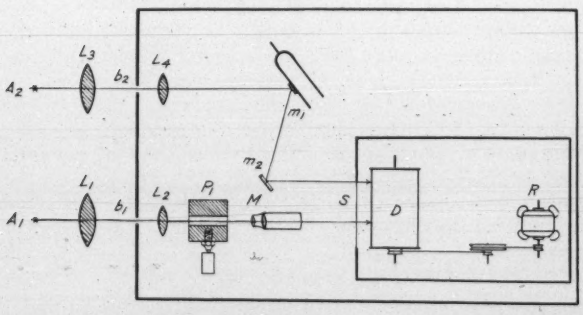
which the permanent indentation is produced, a further experiment will be necessary which will throw light on the mechanical properties of materials beyond the limit of elasticity.

2. EXPERIMENTAL ARRANGEMENT.

It is well known that when a moving sphere impinges on a fixed body, its velocity decreases till it becomes zero, then it reverses its direction, and increases to a certain value. To study the process by which the indentations are produced, it is necessary to obtain the relation between the change of velocity of the impinging sphere and the depth of indentation at every instant of impact.

As the results reported by many authorities show, the duration of the contact of these two bodies is only of an order of 10^{-3} or 10^{-4} seconds, and hence to obtain the record of the motion of the body in this short interval, it is necessary to devise a means by which the whole arrangement shall operate effectively and automatically during the contact. After many unsuccessful trials, we arrived at the following arrangement which worked quite satisfactorily. The plan of the apparatus is shown in Fig. 1.

FIG. 1.

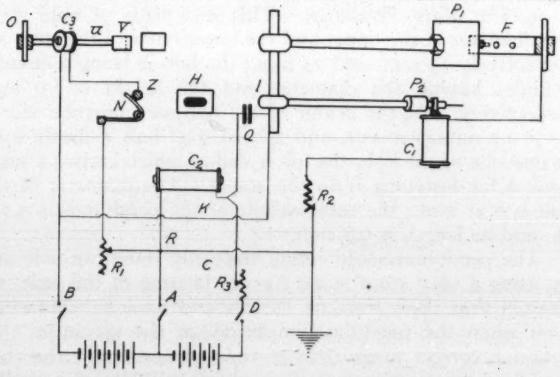


The light from a carbon arc A_1, after passing through lenses L_1, L_2 and a narrow circular aperture b_1, falls on the main pendulum P_1, which carries an impinging sphere made of steel and a small slit measuring 10 × 0.2 mm., and then

enters the magnifying lens M (the magnifications of this lens being about 18) and a photographic system. Inside the camera, there is a rotating drum D with a photographic film placed at the focus of the lens M, so that the motion of the main pendulum can be recorded on the film. A shutter S of a special device was put between the rotating drum and the magnifying lens and so arranged that it only opened during the period of the contact of the impinging sphere and the specimen. The rotating velocity of the drum can be regulated by a small motor and a system of pulleys of different diameters. At the same time, a beam of light from a carbon arc A_2 after passing through lenses L_3, L_4 and a rectangular aperture b_2, is reflected by small mirrors m_1, and m_2 and register the vibrations of the tuning fork on the same film.

In order that all the parts of the apparatus should operate effectively and automatically for the duration of the experiments, the electrical connection as shown in Fig. 2 was

FIG. 2.



devised. The circuit I ($ARCR_2C_1FP_1KBA$) is closed when the main pendulum P_1 is fastened to a staple F; if this pendulum is set free, the circuit is opened and the auxiliary pendulum P_2, the stem of which is made of brass and the bob of soft iron, moves downwards, and a platinum needle

attached to the end of it, closes the circuit II (AC_2HIKBA), as soon as the needle touches the mercury pool H_1 and the coil C_2 attracts the soft iron tips of the shutter. By the choice of a suitable position for the pool and by using the proper quantity of the mercury, the beginning and the duration of the opening of the shutter can be adjusted, the circuit III (AC_2DA) being used for the adjustment of the current to the shutter. Q is a small capacity. The circuit IV ($ARNC_3BA$) serves for the buffer system which prevents more than one impact of the main pendulum on the specimen.

As the sources of the electrical current, we used secondary batteries of 20 volts and by using suitable resistances R_1, R_2 and R_3 the intensities of the current were regulated. Thus by breaking the contact between the main pendulum and the staple by means of some mechanical device, all the parts of the apparatus operated automatically, and records of the impinging sphere were obtained.

We shall next describe some details of the construction of the main parts of the apparatus.

1. *The Main Pendulum.*—This was made of mild steel, the diameter of the upper and the lower parts of the stem are respectively 15 mm. and 25 mm., the bob B being a circular cylinder having the diameter and the height of 70 mm. respectively. At the center of the bob a rectangular slit S, 10×0.2 mm., was cut, and a hard steel ball T firmly fixed at one side of the bob, the lower end of which carries a small hook h for fastening it to the staple. The diameter of the ball is 6.31 mm., the total weight of the pendulum is 3,128 kg. and its length is 505 mm.

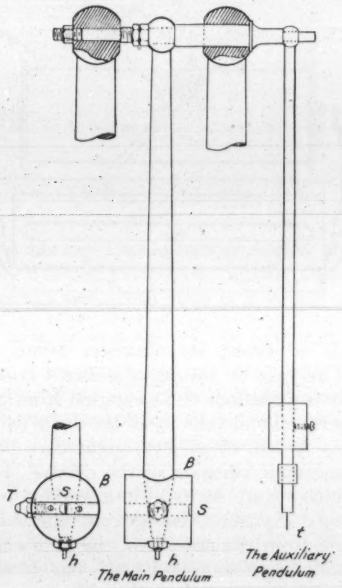
The pendulum could rotate smoothly round an axle and by using a very stout stand for the bearing of this axle, we ensured that there were no flexible and torsional vibrations even when the pendulum impinged on the specimen. To obtain a correct magnitude of the indentation in the steel used as a specimen, we used another pendulum of a similar construction with a stronger stem and a thicker bob.

2. *The Auxiliary Pendulum.*—The stem of this pendulum is made of brass, the bob is of soft iron and, at the lower end of it, is attached a small platinum needle t for the purpose of closing the electrical circuit II. This bob can be moved up

and down along the stem and by varying the period of rotation, we can adjust the time of the beginning and the duration of the opening of the shutter.

3. *The Shutter.*—As shown in Fig. 4, the shutter consists

FIG. 3.

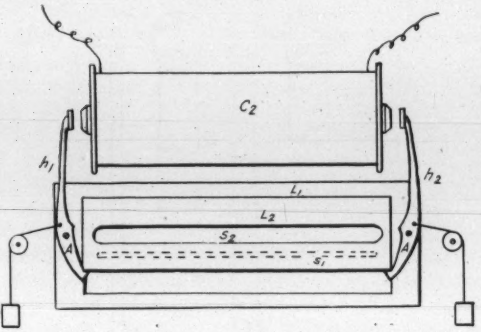


of two brass plates, the one L_1, fixed and having a rectangular slit s_1, 1 × 100 mm., while the other L_2 can move smoothly over the former, having a rectangular slit S_2, 10 × 110 mm. On both sides of the latter, two hooks h_1 and h_2 are provided, which are made of brass with pieces of soft iron at the upper ends of it and can rotate round axes A, A. By passing an electrical current through coil C_2 with a soft iron core, the movable slit L_2 falls down past the fixed slit L_1 and the light from the main pendulum and the magnifying lens falls on

the photographic film at the suitable moment and during the period of contact.

4. *The Rotating Cylinder.*—The rotating cylinder, round which the photographic film is wound, is a hollow circular

FIG. 4.

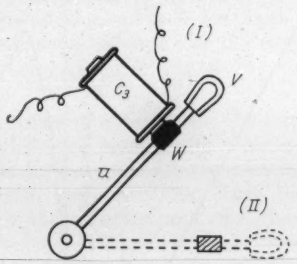


cylinder made of brass, its diameter being 10.5 cm. It rotates round an axis by means of a direct current motor of 0.1 h.p. and its revolutions can be varied from 200 to 400 per minute. The rotational velocity of the drum can be regulated by means of a system of pulleys, great care being taken to ensure the concentric rotation of the drum. The drum and the shutter system were enclosed in a wooden box.

5. *The Buffer System.*—To prevent the rebound of the main pendulum from the specimen, the following device was used. The buffer system was a brass bar U with an india-rubber cap V and soft iron piece W. The bar can be rotated freely about a pin O, near this bar, a fixed bent bar N of brass is in contact with a small sphere Z (Fig. 2) which is one end of the coil C_3, closing the circuit IV, which contains the bent bar and the coil C_3, the coil C_3 attracts the soft iron piece W of the buffer, as shown in the position I in Fig. 5, and if the auxiliary pendulum, after passing through the mercury pool breaks the contact between N and Z, the buffer falls to the position II which prevents the impinging sphere from striking the specimen a second time.

6. *The Tuning Fork.*—The tuning fork used was the usual type driven by a small electromagnet, and its frequency was 200.

FIG. 5.



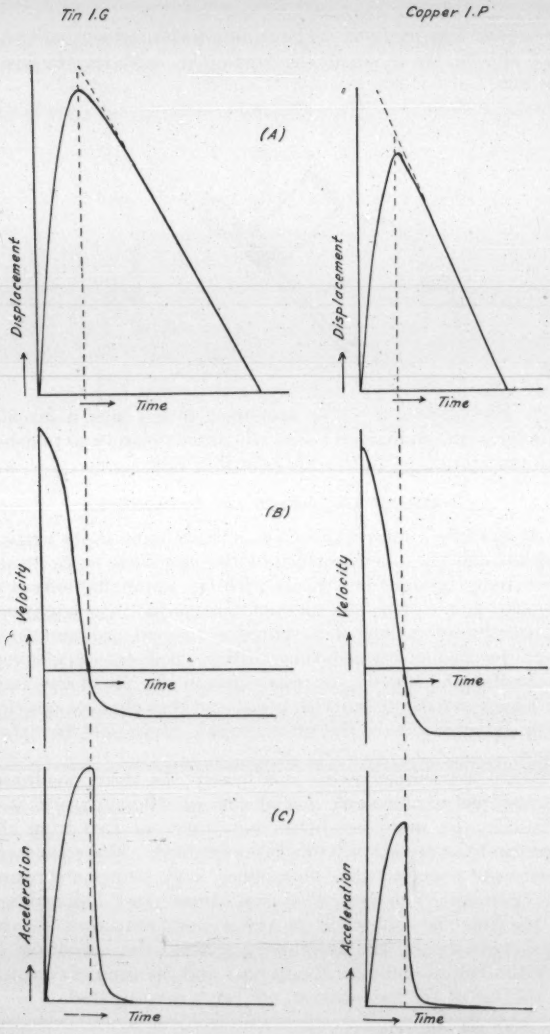
7. *The Specimen.*—The specimen tested was a circular cylinder 4 cm., in diameter, and the impact plane was polished with emery papers and firmly held in a vice.

3. EXPERIMENTAL RESULTS AND CONCLUSIONS.

At first the main pendulum was freely hung in its vertical position and the plane surface of the specimen to be tested was exactly brought in contact with the impinging sphere of the pendulum. The exactness of this contact was confirmed electrically by means of a sensitive tangent galvanometer. When the specimen was in this position, the light was allowed, by opening the shutter, to pass through the slit of the main pendulum and the magnifying glass, and thus the zero position of the indentation on the photographic film could be determined.

Next, the shutter being readjusted, the main pendulum was fastened to the staple and all parts of the apparatus well adjusted; the main pendulum was then set free from the staple and the record of its motion obtained. The specimens tested were cadmium, tin, aluminium, lead, phosphor-bronze, zinc, gunmetal, copper, soft iron, brass and indiarubber. By varying the condition of the experiment, that is, the initial velocity of the impinging sphere, the variation of velocities before and after the impact and the time of contact, the volume of the indentation, etc. were investigated.

FIG. 6.


Tin I.G
Copper I.P
(A)
Displacement
Time
Displacement
Time
(B)
Velocity
Time
Velocity
Time
(C)
Acceleration
Time
Acceleration
Time

The results are summarized as follows:

1. *The Time-displacement Curve and the Time-velocity Curve.*—The records which we have obtained directly by experiments are the time-displacement curve of the impinging sphere, that is the relation between the depth of the indentation and the time, some examples of the curves obtained in our experiment being given in Fig. 6 (*A*) and the annexed plate.

With the exception of indiarubber, the form of the curves for metals are all similar in character. Since the tangent at any point of this curve represents the velocity at that point of the indentation, we can obtain from these curves the relation between the velocity and the time as shown in Fig. 6 (*B*).

From these two figures, it can be concluded that the rate of the penetration is at first rapid and then gradually decreases. In other words, the rate of the decreases of the velocity of the impinging sphere is comparatively small in the first stage of penetration, but becomes greater in the last stage. After the maximum indentation has been reached, the direction of the motion of the sphere is reversed and its velocity gradually increases until it finally attains a constant value. The fact that the sphere regains some of the velocity, shows that some parts of the indentations elastically recover after the sphere has ceased to penetrate.

The velocity of the sphere increases as this recovery progresses, becoming finally a constant value.

It is therefore a very interesting question whether the forms of the indentation remain spherical or not. Experiments to solve this problem, by varying the materials both of the specimen and the impinging spheres, are now in progress and the result will be published in the near future.

2. *The Force-time Curves.*—Again differentiating the velocity-time curves mechanically, we obtain the force-time curves as shown in the Fig. 6 (*C*). The force in the contact surfaces increases at first slowly and then rapidly to a maximum value at the instant when the greatest indentation is produced and then decreases to zero. This result coincides with those obtained by H. Seehase [2] in his experiment on

[2] H. Seehase, *Forschungsarbeiten*, Vol. 182, 1915.

the deformation of materials which were struck by a falling body and that of A. Elmendorf's [3] timber break experiment, but in contrast to that of R. Planck [4] in the case of the extension of materials by a sudden loading. It will be observed that this method of measuring force requires two differentiations of the displacement-time curves and hence there may be a certain degree of ambiguity in the magnitude of these forces. But it is certain that the maximum of the force corresponds to the maximum depth of the indentation, in this respect, the differentiating instrument designed by Elemendorf is very valuable, but the direct measurement of the force is desired.

3. The relation between the volume of indentation and the loss of kinetic energy of the impinging sphere.

According to the results previously published, there exists approximately a proportional relation between the volume of the indentations remaining after the impact and the total loss of the kinetic energy of the impinging sphere. It is very interesting to find whether the same relation holds good at every instant of the impact.

TABLE I.

V(mm.³).	E(× 10⁶ erg).					
	Al.	Sn.	Zn.	Cu.	Brass.	Soft Iron.
0.185	0.038	0.143	0.221	0.450	0.542	0.948
0.795	0.163	0.296	0.954	0.194	2.35	3.95
1.74	0.358	0.859	2.05	4.32	5.13	8.94
3.10	0.639	1.534	3.66	7.59	9.12	15.92
4.40	....	....	....	....	....	22.54
4.58	....	....	....	....	13.54	
4.75	0.974	2.33	5.68	11.65		
5.98	1.22	2.89	....			
6.10	....	....	7.34			
6.74	1.40	....				
7.55		3.68				
7.59		3.76				

From the displacement-time and the velocity-time curves, we can calculate the total loss of the kinetic energy E and

[3] A. Elmendorf, JOURN. FRANK. INST., Vol. 182, p. 771, 1916.

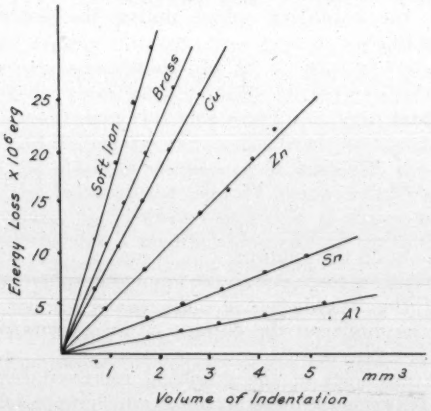
[4] R. Planck, *Forschungsarbeiten*, Vol. 133, 1913.

the volume of indentation at every instant of penetration; the result shows that a proportional relation approximately exists between them, and the constant of proportionality is different for different materials as shown in the table I, II, and Fig. 7. Thus it is concluded that the impinging sphere penetrates the other body so as to make the volume of the indentation proportional to the loss of the kinetic energy up to that instant. It is very interesting that this relation holds good not only for the volume remaining after the impact, but also for all the stages during the impact.

TABLE II.

	Pb.	Al.	Sn.	Zn.	Cu.	Phosphor Bronze.	Brass.	Soft Iron.
Energy Loss $\frac{10^6 \text{ erg}}{\text{mm.}^3}$	0.074	0.205	0.512	1.21	2.44	2.78	2.95	4.18

FIG. 7.



As Lord Rayleigh [5] has pointed out, the dissipation of energy during the impact is due to two causes: (*a*) the

[5] Lord Rayleigh, *Phil. Mag.*, Vol. 11, p. 283, 1906.

production of sound, (*b*) the strain caused during an inelastic collision The former is very small in comparison with the latter as Banerji [6] has shown, and if we leave this energy out of consideration, the dissipation of energy has to be ascribed entirely to the formation of the indentation. Thus the above result also confirms this conclusion.

Next, consider the resisting force of a body subjected to an impact. If we denote the mass of the impinging sphere by M, the velocity just before the impact and that at the instant when the depth of the indentation is x, by v_0 and v respectively, and the diameter of the circle of the indentation by y, the above mentioned conclusion can be expressed by the equation

$$K\int_0^x \pi Y^2 dx = \frac{1}{2}M(V_0^2 - V^2),$$

where K is the constant specified by the materials of the specimens.

Assuming that the resisting force is a function of the depth of the indentation and denoting it by $f(x)$, the work done by the impinging sphere during the penetration is expressed by

$$\int_0^x f(x)dx$$

and we have

$$\int_0^x f(x)dx = K\int_0^x \pi y^2 dx = \frac{1}{2}M(v_0^2 - v^2)$$

$$\therefore \quad f(x) = K\pi y^2.$$

Thus we obtain the following conclusion:

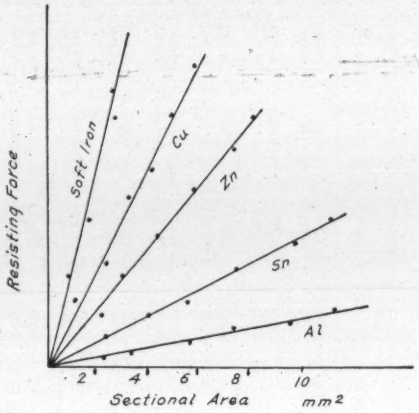
The resisting force due to the impinging sphere at a given depth of the indentation is proportional to the sectional area of the circle on the surface of the specimen at that instant.

Some examples of the graphical representation of the relation between the resisting force which were taken from Fig. 6 (*C*) and the calculated value of the area of the circle on the surface of the specimen at every instant of the indenta-

[6] S. Banerji, *Phil. Mag.*, Vol. 31, p. 96, 1916, and Vol. 35, p. 95, 1918.

tion are shown in Fig. 8, these curves confirming the above conclusion.

FIG. 8.



4. *The Time of Contact.*—The time of contact of the two bodies is next considered. As M. G. Moreau [7] has pointed out, there are three phases during impact; namely (1) the phase of penetration τ_1 during which the ball penetrates the material and its velocity diminishes from the initial value to zero, (2) the phase of reaction τ_2 during which the material react on the ball and the velocity increases from zero to a certain value, (3) the phase of release τ_3 during which the ball passes through a superficial layer of air up to the initial position, its velocity remaining constant.

From the displacement-time curve, we can evaluate directly the time intervals corresponding to these three phases; the results were as follows: (1) the total time of contact decreases as the velocity of the ball before the impact increases; (2) the phase of the reaction is the shortest, while the third phase is the longest of all; (3) the ratios of these three intervals are nearly constant for a given material, and independent of the initial velocity. This last result can

[7] M. G. Moreau, *Journ. d. Phys. et Radium*, Tome 11, p. 329, 1921.

TABLE III.

	Initial Velocity cm./sec.	τ_1 (sec.).	τ_2 (sec.).	τ_3 (sec.).	$\tau_2 : \tau_1$.	δ_1 (mm.).	δ_2 (mm.).	$\delta_2 : \delta_1$.	e.
Sn	168	13×10^{-4}	8×10^{-4}	46×10^{-4}	0.61	0.71	0.07	0.10	0.11
	174	12	8	47	0.67	0.82	0.08	0.10	0.11
Al	174	14	8	47	0.57	0.78	0.09	0.113	0.208
	174	13	7	48	0.54	0.86	0.10	0.116	0.209
Pb	168	21	10	123	0.47	1.86	0.10	0.054	0.083
	174	20	9	127	0.45	1.67	0.09	0.055	0.084
Zn	102	9	4	14	0.45	0.73	0.14	0.141	0.157
	124	8	4	15	0.50	0.73	0.14	0.141	0.155
Cu	102	7	4	16	0.57	0.75	0.10	0.134	0.313
	111	5	3	11	0.60	0.70	0.09	0.129	0.320
Soft Iron	124	4	3	8	0.75	0.63	0.10	0.158	0.373
	150	3	3	6	100	0.69	0.11	0.158	0.365
Brass	102	5	3	10	0.60	0.72	0.10	0.140	0.340
	111	7	4	8	0.57	0.73	0.10	0.140	0.340

be inferred from the fact that the ratios between the amounts of the penetration δ_1 and of restitution δ_2 are nearly constant for a given material. This is equivalent to the fact that the time of contact and the coefficients of restitution are nearly constant for different velocities beyond a certain value.

5. The results obtained in the case of indiarubber. In the previous investigation, it was noted that the behavior of indiarubber when subjected to impact, is somewhat different from the metals. The coefficient of restitution of the former is large as in the case of iron and copper, but the duration of contact is as long as one hundred times that of the latter. While, in the case of metal the shorter the duration of contact, the larger the coefficient of restitution.

In the present experiment also, the displacement-time curve and the velocity-time curve of indiarubber are somewhat different from those of metals as seen from Fig. 9. The characteristic point of the displacement-time curve is that no straight portion of the curve is observable in the third phase, and hence it seems probable that the sphere is accelerated beyond the zero line of contact. Thus we conclude that the specimen is put in a state of vibration by the local application of the impulsive force.

Since in the case of indiarubber, no indentation remains after the impact, it is clear that the energy consumed during the impact is mainly used up in the vibration of the specimen. Even in the case of metals, some energy will also be used in the vibration of the specimen, but it will be very small in comparison with that consumed in the indentation.

SUMMARY.

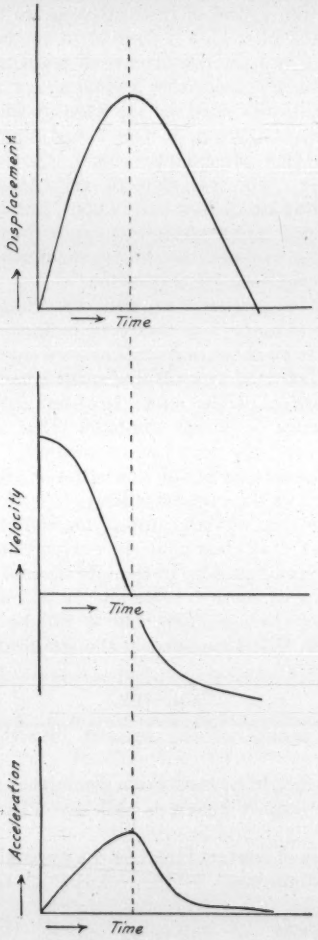
The main results of the present investigation are as follows:

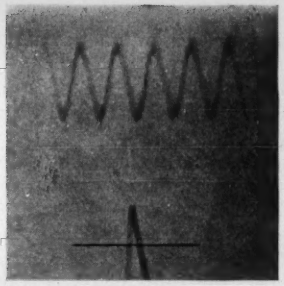
(1) The coefficient of restitution diminishes as the velocity just before the impact increases and beyond a certain value of this velocity, it diminishes linearly.

(2) The time of contact between the impinging sphere and the specimen diminishes as the velocity of the impact increases.

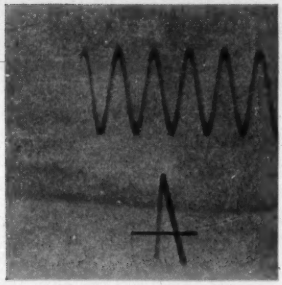
(3) In the case of impinging sphere, the volume of

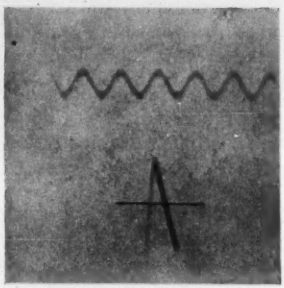
FIG. 9.



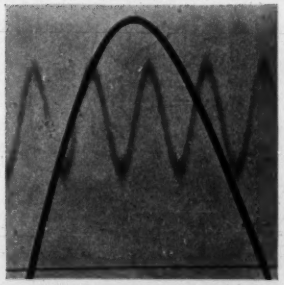


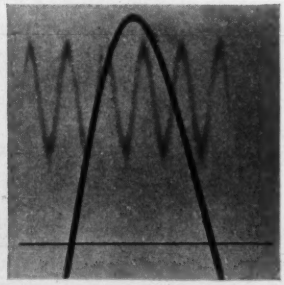
Brass. Initial vel. = 110 cm./sec.; magnif. = 13.8.
Soft iron. Initial vel. = 150 cm./sec.; magnif. = 13.8.





Zinc. Initial vel. = 102 cm./sec.; magnif. = 13.8.
Copper. Initial vel. = 150 cm./sec.; magnif. = 13.8.





Indiarubber. Initial vel. = 168 cm./sec.; magnif. = 13.8.
Indiarubber Initial vel. = 161 cm./sec.; magnif. = 13.8.

indentation is proportional to the total loss of kinetic energy up to that instant.

(4) The penetrating sphere is subjected to a resisting force which is proportional to the area of the circle on the surface of specimen at the instant of indentation.

(5) There are three phases of impact, the time of indentation, that of reaction, and that of release, among which the time of reaction is the shortest, and the time of release is the longest of all. The ratios of these three phases are nearly constant for a given material.

(6) In the case of indiarubber, the third phase is completely lacking and the impinging body is subjected to a further acceleration beyond the zero line of contact.

In conclusion, the writers express their hearty thanks to the Saito Gratitude Foundation, for one part of the expenses of the present experiment which was paid from the research fund of the Foundation.

A DETERMINATION OF A LIMIT OF THE NUMBER OF FREE ELECTRONS IN A METAL.*

BY

F. EVELYN COLPITTS.

THE original theories of metallic conduction of Riecke,[1] Drude[2] and J. J. Thomson[3] considered both thermal and electrical conduction as due to free electrons existing in metals with energy $\alpha\theta$ and obeying the ordinary gas laws. These simple theories have been found to explain certain facts such as the Wiedemann-Franz Law but have failed in many phenomena such as the Thomson effect, superconductivity and atomic heats. Other theories have gradually replaced the electron theory of metallic conduction, but recently Sommerfeld[4] has restored it by the use of the Fermi-Dirac statistical method. The concentration of free electrons varies for the different theories from 3 to 10^{-5} times the number of atoms.

The number of free electrons is here found from direct observations following a suggestion in J. J. Thomson's Corpuscular Theory of Matter. Although its practical application has necessitated considerable departure from the plan and theory of the experiment as there suggested, the basic principle holds.

On the free electron theory the charge on a body is the result of an excess or deficiency of free electrons. Ordinary electrostatics considers this charge as residing on the surface only. But, since the electrons are all negatively charged, they repel one another and consequently there is a finite thickness through which they diffuse due to the electronic pressure. The detailed theory shows that a difference of

* Part of a dissertation presented for the degree of Doctor of Philosophy in Yale University.

[1] E. Riecke, *Wied. Ann.*, *66*, 353 u. 545, 1898.

[2] P. Drude, *Ann. d. Physik*, 306, *1*, 566, 1900.

[3] J. J. Thomson, Rapports du Congres de Physique de 1900, Paris, *3*, p. 138.

[4] A. Sommerfeld, 'Naturwiss,' p. 825, 1927.

potential exists between any two points in the metal which correspond to different concentrations of electrons.

These considerations may be applied to the case of a thin film of metal on the inside of a glass cylinder. Suppose that the ends of the cylindrical film are continued into heavy collars of the metal and that an electric field is applied between the thin film and the layer of tinfoil surrounding the cylinder. Then, if the film is sufficiently thin, a portion of the charge density will penetrate to the inside and there will be a difference of potential between the inside of the film and the heavy collars. This difference of potential is found to be a function of n, the number of free electrons per unit volume. Hence either a definite value of n or, if a zero result is obtained for the potential difference to be measured, a definite lower limit for n may be determined.

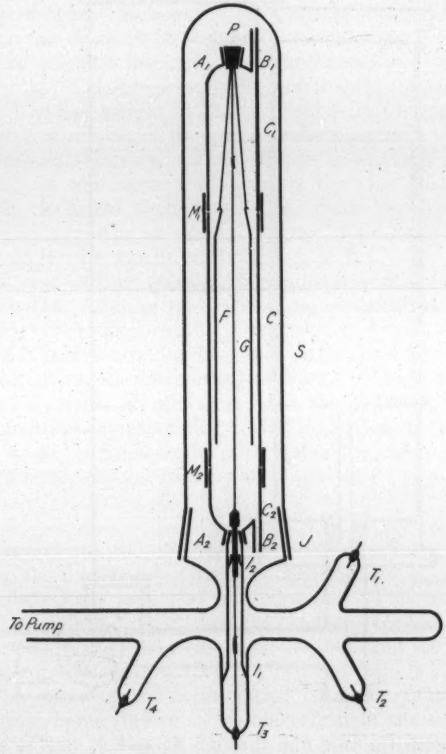
APPARATUS.

Fig. 1 gives the apparatus used for depositing the films. A large pyrex glass cylinder S rests on a ground glass joint J supported by a wooden frame. An inner seal I_1 supports a smaller tube ending in a ground glass joint A_2. Resting on A_2 is a glass cylinder C_2 and above it the glass cylinder C on which the film is to be deposited. Above C is another cylinder C_1 ending in the ground glass joint A_1. The parts are held together and aligned by iron wires passing around C_1, C, C_2 and screwed to M_1 and M_2, two metal collars fitting closely on the junctions of C with C_1 and C_2.

P is a steel plug resting in A_1. The glass guard G is hung from P by nickel wires, while the heavy collars are being deposited. The filament F is supported by a coiled copper wire from the center of P. The lower end is held by a pin vise passing through A_2 and held centrally by an inner seal I_2. Another coiled copper wire connects the pin vise to the tungsten seal T_3. When the apparatus is used for evaporating the films, P is connected to T_4. Fig. 1 gives the set-up for sputtering or evaporating the heavy films of metal on the glass cylinder C. For depositing the thin part of the film, the guard G was removed. For the purpose of measuring the resistance of the film during deposition, copper wires were soldered to the heavy films, passing through B_1 and B_2

and connected to T_1 and T_2 respectively. The whole apparatus was connected to a customary vacuum system consisting of a two stage mercury diffusion pump backed by a Cenco

FIG. 1.

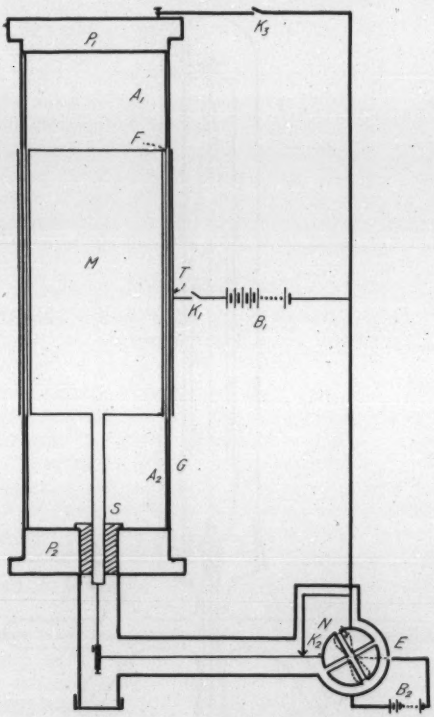


Hyvac Oil pump. Liquid air was always used during deposition of the films.

Fig. 2 shows the apparatus used for performing the main

experiment. F is the thin film deposited on the glass cylinder G, having heavy films A_1 and A_2 on the ends. P_1 and P_2 are brass discs which fit tightly in the glass cylinder and make

FIG. 2.



contact with the thin film through A_1 and A_2. M is a hollow brass cylinder with solid ends supported by a heavy rod through the sulphur plug S and connected to the quadrant electrometer E. T is a sheet of tinfoil wrapped around G and connected to the high potential battery B_1. N, the

needle of the electrometer, is connected to one pole of the constant potential battery B_2.

PROCEDURE AND RESULTS.

The thickness of the film was obtained by two methods:

(1) By weight. Either the increase in weight of the cylinder or the decrease in weight of the filament gives the thickness of the film assuming the same density as for the metal in bulk. These two methods agreed within 5 per cent. for gold. The first weight method did not work for tungsten.

(2) By comparison of its specific resistance with that of films of known thickness. This method is justified by the experiments of Mackeown [5] who shows that for films comparable in thickness with those here used, the apparent specific resistance has its normal value. This method was also checked by the weight method.

A 1000 volt battery was used for sputtering in the case of gold and a D.C. heating current for the evaporation of the tungsten.

When a film appeared uniform and had a resistance of the right value, it was used according to Fig. 2. Then, with K_3 closed and K_2 open, K_1 was closed and the deflection noted.

The potential corresponding to the deflection of the electrometer is not the same as the potential of the inside surface of the film but is related to it by the equation

$$V/P = - q_{yx}/q_{yy},$$

where V represents the potential on the electrometer system by raising the film to a potential P, and q_{yx} and q_{yy} are the usual coefficients of induction and capacity. This ratio was found by applying P directly to the electrometer and noting the deflection θ_2, then applying it to the film and noting the deflection θ_1 corresponding to V. Then the ratio is given by θ_1/θ_2. The value of the ratio varied between .71 and .79, the variation being due to slight differences in the set-up of the parts M and F, Fig. 2.

In the case of Gold No. 1, where a deflection was obtained, a test for actual holes in the film was made by using, instead of the sheet of tinfoil T, Fig. 2, a narrow band of copper. If

[5] S. Mackeown, *Phys. Rev.*, *23*, 85, 1924.

there were actual holes in the film, there should be certain positions of the copper band at which the electrometer would deflect suddenly. No such places were found. The tungsten film oxidized so rapidly that it was not tested in this way, but no holes or scratches could be noticed on it.

The resistance of the film played no part provided that it was not great enough to produce a difference of potential along the film due to the leakage through the glass. The conductivity of the glass was measured and the conduction current was only 10^{-8} amperes using 4000 volts. In the case of the gold films, the resistances were 10 and 16 ohms respectively. The resistance of Tungsten No. 1 was 150,000 ohms which might give a drop of potential along the film of 1.5×10^{-3} volts, but the measured change of potential of the film on applying the potential to the tinfoil was 4.8×10^{-2} volts. Hence no great part of it could be due to the high resistance of the film. The resistance of Tungsten No. 2 was only 10,000 ohms. In no case would the effect of the resistance explain the deflection obtained. Table 1 gives the actual results.

TABLE 1.

Substance.	Potential Applied volts.	Electrometer Sensitivity mm/volt.	q_{xy}/q_{yy}.	Film Thickness cm $\times 10^{-8}$	Average Deflection mm.	ΔV e.s.u.
Gold No. 1.	3,280	1,334	.71	2.0	4.0	1.4×10^{-5}
"	2,000	1,334	.71	2.0	2.5	8.8×10^{-6}
Gold No. 4.	1,030	1,550	.78	1.5	0.0	$<2.8 \times 10^{-6}$
"	2,700	5,300	.78	1.5	0.0	$<8.1 \times 10^{-7}$
"	2,860	5,550	.78	1.5	0.0	$<7.7 \times 10^{-7}$
Tungsten No. 1	4,000	480	.79	7.5	24.0	2.1×10^{-4}
"	4,000	1,500	.79	7.5	59.0	1.7×10^{-4}
No. 2.	4,000	1,040	.79	9.9	0.0	$<4.1 \times 10^{-6}$
"	4,000	6,220	.79	9.9	0.0	$<6.8 \times 10^{-7}$
"	4,000	15,000	.79	9.9	0.0	$<2.8 \times 10^{-7}$

Thickness of glass cylinder = 1.2 mm.
< gives the value corresponding to 1 mm. deflection.

THEORY AND INTERPRETATION OF RESULTS.

Let n = the normal number of free electrons per cc. in a certain metal,

p = the electronic pressure at any point,

X = the electric field in any direction,

ξ = the increase in number per cc. at any point over the normal number

Then $p = RT(n + \xi)$ at any point and

$$\frac{dp}{dl} = RT\,\frac{d(n+\xi)}{dl} \text{ in any direction characterized by } dl. \quad (1)$$

Also

$$\frac{dp}{dl} = -(n+\xi)e\frac{dV}{dl}, \quad (2)$$

giving

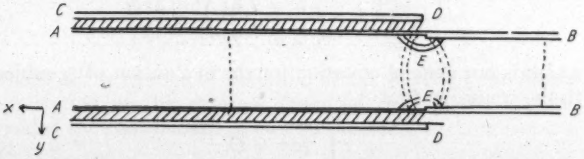
$$dV = -\frac{RT}{e}\,\frac{d(n+\xi)}{(n+\xi)}$$

or

$$V_2 - V_1 = -\frac{RT}{e}\log(n+\xi)\Big|_1^2. \quad (3)$$

Suppose this general equation is applied to the case of a thin sheet, or film of metal, AB, separated from another sheet CD by a dielectric, Fig. 3, CD is supposed to be connected

FIG. 3.



to a high potential battery, the other end of which is connected to AB. Then Eq. (3) will give the value of ΔV of the difference of potential between EB and any point in AE when the number of free electrons at that particular point is changed by the presence of CD. The field exists only near E, the junction of the thin part and the thick part, hence not far beyond this junction, both on the thin and the thick side, the equipotential surfaces will be infinitely far apart. The lines of force are represented by heavy lines and the equipotential surfaces by dotted lines.

We will then have

$$\Delta V = -\frac{RT}{e}\log\frac{n+\xi}{n} \quad (4)$$

since n = the electronic concentration in EB

$n + \xi$ = the electronic concentration in AE at the point considered.

The problem now reduces to that of finding ξ on the inner surface of the thin film when the thickness (η) of the film is known.

By Gauss' Theorem

$$\frac{d^2V}{dy^2} = -4\pi\xi e;$$

but from (3)

$$\frac{dV}{dy} = -\frac{RT}{e}\frac{d}{dy}\log(n+\xi);$$

hence

$$4\pi\xi e = \frac{RT}{e}\frac{d^2}{dy^2}\log(n+\xi)$$

or

$$\frac{1}{n+\xi}\frac{d^2\xi}{dy^2} - \frac{1}{(n+\xi)^2}\left(\frac{d\xi}{dy}\right)^2 = \frac{4\pi\xi e^2}{RT}, \tag{5}$$

which is our general equation for the evaluation of ξ, subject to the condition that

$$e\int_0^{\eta}\xi dy = Q, \tag{6}$$

where Q is the charge per unit area on AE.

Integrating equation (5), we obtain

$$\frac{\left(\frac{d\xi}{dy}\right)^2}{(n+\xi)^2} = \frac{8\pi e^2}{RT}(\xi - n\log(n+\xi) + C_1). \tag{7}$$

If it were true that $d\xi/dy = 0$ when $\xi = 0$, we could write $C_1 = n\log n$. This assumption would be invalid without further examination in the present instance, however; and, in general we must write

$$C_1 = n(\log n + C_3). \tag{8}$$

Equation (6) may then be written

$$\frac{\frac{d\xi}{dy}}{n+\xi} = \left(\frac{8\pi e^2}{RT}\right)^{1/2}\left(\xi - n\log\frac{n+\xi}{n} + nC_3\right)^{1/2}.$$

If the transformation $s = \log(n+\xi)/n$ is made, it becomes

$$\frac{ds}{dy} = (ne^s - n - ns + nC_3)^{1/2}\left(\frac{8\pi e^2}{RT}\right)^{1/2},$$

$$\frac{ds}{dy} = \left(\frac{8\pi e^2 n}{RT}\right)^{1/2}(e^s - 1 - s + C_3)^{1/2}, \qquad (9)$$

so that

$$\int_{s_1}^{s_2}\frac{ds}{(e^s - 1 - s + C_3)^{1/2}} = \left(\frac{8\pi e^2}{RT}\right)^{1/2} n^{1/2}(y_2 - y_1). \qquad (10)$$

Now, measuring y from the inside of the film, let $y_1 = 0$ correspond to s_1. Then, since $s = \log(1 + \xi/n)$, equation (4) yields

$$s_1 = \log\left(1 + \frac{\xi_1}{n}\right) = -\frac{\Delta Ve}{RT},$$

where ΔV is change of potential of the inside of the film on application of the external potential, and is the quantity yielded by experiment. Thus (8) becomes

$$\int_{-(\Delta Ve/RT)}^{S}\frac{ds}{(e^s - 1 - s + C_3)^{1/2}} = \beta n^{1/2}y + C. \qquad (11)$$

If we assign a tentative value of C_3, it is possible to evaluate the left hand side graphically as a function of its upper limit S. Thus, it becomes possible to plot S against $\beta n^{1/2}y + c$ for this tentative value of C_3. Since $S = \log(1 + \xi/n)$, it thus becomes possible to plot ξ/n against $\beta n^{1/2}y + c$ for the assumed C_3. Now the total charge Q per unit area of the film is given by

$$Q = e\int_0^\eta \xi dy$$

or, changing the variable

$$\frac{\beta Q}{en^{1/2}} = \int_C^{\beta n^{1/2}\eta + C}\frac{\xi}{n}d(\beta n^{1/2}y + c). \qquad (12)$$

Since, for the assumed C_3, we have ξ/n as a function of $\beta n^{1/2}y + C$, it is possible to plot ξ/n against $\beta n^{1/2}y + C$. If, still corresponding to the tentative value of C_3 we choose a tentative value of n, it is possible to find what value must be taken for C in order that when the integral on (8*b*) is graphically evaluated over the range C to $\beta n^{1/2}y + C$, the result so obtained is equal to the left hand side $\beta Q/en^{1/2}$. The value of ξ/n at the upper limit of the integral can then be found; and, if n has been chosen properly in relation to C_3, it should result that the use of the upper limit for ξ/n in (4) will give for ΔV a value equal to that yielded by experiment. If it does not, the procedure is to choose another value of n for the same C_3 and proceed in this way by interpolation until the correct value of n is found. In this manner, a number of possible n's corresponding to a various assigned C_3's could be obtained. The problem which remained was that of fixing C_3. How is this to be done?

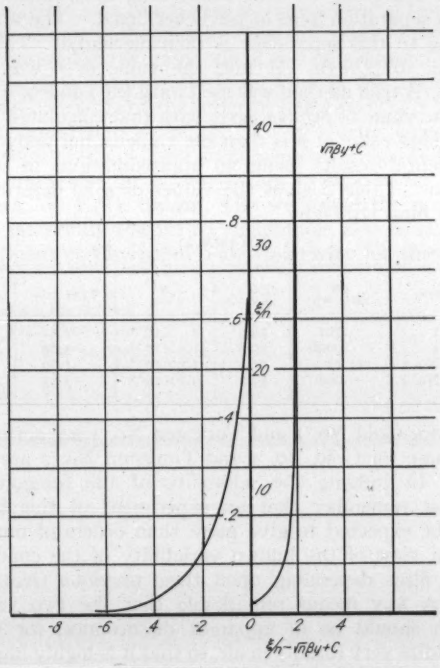
Now it will be observed that although equation (4) gives ΔV (i.e., $V_\eta - V_0$) quite explicitly as a function of ξ/n at $y = \eta$, it does not tell us that ΔV is entirely due to the charge on the film. Thus, for example, if concentrically with the cylinder, there had been a charged wire, the quantity $V_\eta - V_0$ would have been determined in part by the distribution of charge throughout the film and in part by the charge on the wire. Put in general terms the actual value of $V_\eta - V_0$ would in general be composed of two parts, a part calculable directly from the charge distribution throughout the film, and a part which was a solution of Laplace's equation and which would correspond to charged bodies outside the film. In the present instance, the only charged body outside the film, is the foil cylinder around the outside of the glass; and, since this produces no field in its interior, our $V_\eta - V_0$ is limited to the value calculable from the charge distribution on the film alone.

In other words, we must have

$$V_\eta - V_0 = -\int_0^\eta 2\pi dy \int_0^y \xi e dy$$
$$= -\frac{2\pi e n^{1/2}}{\beta}\int_0^\eta dy \int_{\varepsilon}^{\beta n^{1/2}y + C} \frac{\xi}{n} d(\beta n^{1/2}y + C). \qquad (13)$$

Now for every value of C_3 chosen, we have a corresponding value of n and of C. Moreover, the graphs for evaluating the integral with C in the limits are the same as those used in connection with equation (8*b*), so that it is possible to evaluate this integral as a function of y and so evaluate, again graphically, the integral whose limits are zero and η. If the result is not equal to $V_\eta - V_0$ as measured experimentally, the assumed value of C_3 was wrong. We must then choose the value of C_3 whose corresponding n and C, chosen as they have been to satisfy (8*b*), also satisfy (8*c*). The corresponding value of n is then the value sought.

FIG. 4.



Proceeding in the above manner it was found that for gold film No. 4, and for the tungsten films, $C_3 = 0$. Fig. 4 shows the curve for ξ/n plotted against $\beta n^{1/2}y + c$ for the case $C_3 = 0$. By using the foregoing method, it is possible to show that for values of $n = 10^{17}$ for gold, the value of ΔV to be expected is 1.8×10^{-4} volt. The limiting value of ΔV experimentally observable was taken as 10^{-4} volt. Hence, the conclusion is that for gold, n must be greater than 10^{17} electrons per cc.

A simpler method is to find from Eq. (4) the value of ξ/n corresponding to the experimentally determined value of ΔV. Then the value of $\beta\sqrt{n}\eta + C$ is read directly from the graph. Fig. 4. Assigning a value to n, C is determined and the abscissa separation $\beta\sqrt{n}\eta$ is, moreover, fixed. The area corresponding to this separation is then measured. This should be equal to $\beta Q/e\sqrt{n}$. If not, the value of n was wrongly chosen. A trial method was used until the value of n assigned made the value of $\beta Q/e\sqrt{n}$ agree with that calculated from the area. This value of n is then the value in harmony with the experimental results using no approximations in its determination. Table 2 gives the values of n so determined for the four films studied.

TABLE 2.

Substance.	ξ/n (at $y =$).	Q e.s.u.	η cm.	$\beta\sqrt{n}\eta + C$.	n per c.c.
Gold No. 1	.101	26.5	2×10^{-6}	-2.32	3.6×10^{16}
Gold No. 4	.0096	35.8	1.5×10^{-6}	-5.65	$> 1.6 \times 10^{17}$
Tungsten No. 1.	11.18	53.1	7.5×10^{-6}	1.80	1.4×10^{15}
Tungsten No. 2.	.008	53.1	9.9×10^{-6}	-5.85	$> 9.2 \times 10^{15}$

Results for Gold No. 1 and Tungsten No. 1 are actual values while those for Gold No. 4 and Tungsten No. 2 are limiting values. In judging the reliability of the foregoing data, one must remember that an experiment of this kind can hardly be expected to give more than orders of magnitude. Then, in view of the known variability of the conductivity of thin films depending upon their previous treatment, it is not by any means remarkable that the two values for tungsten should be in apparent discordance, for tungsten films oxidize very rapidly in air, so that it is highly improbable that films 1 and 2 were in similar conditions.

If $\xi << n$ as assumed by J. J. Thomson, Eq. (6) reduced to

$$\Delta V = \frac{4\pi Q}{p} \frac{e^{-ph}}{1 - e^{-pq}} \quad \text{if} \quad p = \left(\frac{4\pi e^2 n}{RT}\right)^{1/2},$$

i.e., the difference of potential between any point $y = h$ and EB. But by using the detailed theory it is found that for values of $n = 10^{17}$ per cc. this approximation cannot be used, hence the graphical method must be used.

CONCLUSION.

The difference of potential between two points of a metallic conductor, having different concentrations of free electrons, was calculated as a function of n, the number of free electrons per unit volume in the metal. This difference of potential has been measured in two cases and in two others found to be zero. From the data obtained the lower limit of the number of free electrons in gold is of the order of magnitude of 2×10^{17}/cc., and in tungsten 9×10^{18}/cc. These results were obtained with a sputtered gold and an evaporated tungsten film.

In conclusion the author wishes to express her thanks to Professor W. F. G. Swann, who suggested the problem and helped materially toward its completion, also to many of the Department of Physics of Yale University for their coöperation.

SLOANE LABORATORY,
YALE UNIVERSITY,
May 31, 1928.

ERRATA.

AN EXPERIMENTAL INVESTIGATION OF FORCED VIBRATIONS.

BY

L. W. BLAU, M.A.

September, 1928, page 355.

Equation 5, page 557, should read

$$y(t) = \Big\{A^2 + B^2 + (C^2 + D^2)e^{-2Rt} - 2e^{-Rt}\big[(A^2 + B^2)(C^2 + D^2)\big]^{1/2} \sin\left[(s - \Delta n)t + \tan^{-1}\frac{C}{D} + \tan^{-1}\frac{B}{A}\right]\Big\}^{1/2} \sin\,(pt + \beta)$$

where

$$\beta = \tan^{-1}\frac{A + e^{-Rt}(C^2 + D^2)^{1/2} \sin\left[(s - \Delta n)t + \tan^{-1}\frac{C}{D}\right]}{B + e^{-Rt}(C^2 + D^2)^{1/2} \cos\left[(s - \Delta n)t + \tan^{-1}\frac{C}{D}\right]}$$

Fig. 21, page 371, should be Fig. 22 and Fig. 22 should be Fig. 21.

WHAT IS THE MAGNETIC PERMEABILITY OF IRON?

BY

T. D. YENSEN, Ph.D.

ABSTRACT.

This paper is a review of the changes brought about in the magnetic properties of "iron" during the period 1870 to 1928 and shows the absurdity of using "iron" as a standard for comparison. The latest (1928) value for the initial permeability (μ_0) of "iron" is given as 1150, its maximum permeability (μ_{max}) as 61,000, and its hysteresis loss (W_h) as 300 ergs per cubic centimeter per cycle for B = 10,000 gausses. The corresponding values prior to 1900 were: $\mu_0 = 250$ $\mu_{max} = 2600$, $W_h = 3{,}000$.

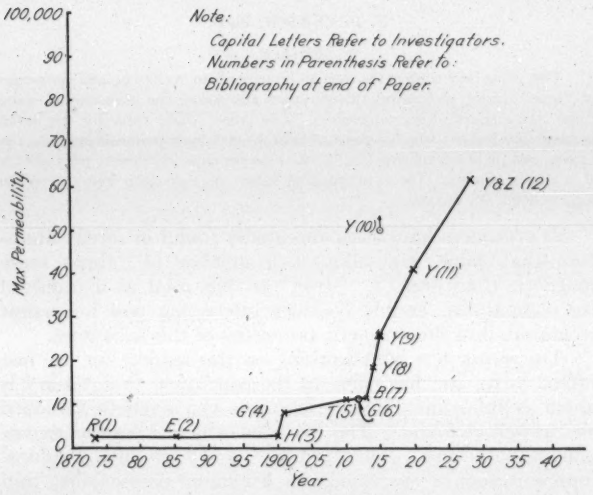
STATEMENTS have been repeatedly found in recent literature that some new alloy is a number of "times more magnetic than iron." * "Iron" is thus used as a standard for comparison, and it becomes interesting and important to inquire into the magnetic properties of this substance.

The writer has been working on this subject for the last fifteen years and has come to the conclusion that "iron" is about as unfortunate a material to use as a magnetic standard as can well be found. This becomes evident from the accompanying table and a glance at Figs. 1 and 2, giving a graphical representation of the change in maximum permeability and hysteresis loss of "iron" in the purest forms obtainable, during the period 1870 to date. When Ewing in 1885 thus talked about the magnetic properties of "iron" he mentioned a maximum permeability of around 2000,[1, 2, 3] which was the value imprinted on the writer's mind during his school days in the 90's. Between 1900 and 1910 there came new activity and interest in magnetic materials with the advent of silicon steel and electrolytic iron, and the maximum permeability of "iron" was gradually pushed to 10,000.[4, 5, 6, 7] Between 1910 and 1920 the writer, by taking additional precautions to prevent contamination of the electrolytic iron and to eliminate impurities, succeeded in increasing the maxi-

* See for example: *Science*, Vol. 57, p. 526, May 25, where this statement is found: ". . . permalloy, a new alloy of nickel, 100 times more magnetic than iron."

mum permeability of "iron" first to 19,000,[8] then to 25,000[9] and to 40,000,[11] and by the addition of small amounts of silicon (0.15 per cent.), obtained values greater than 50,000.[10]

FIG. 1.

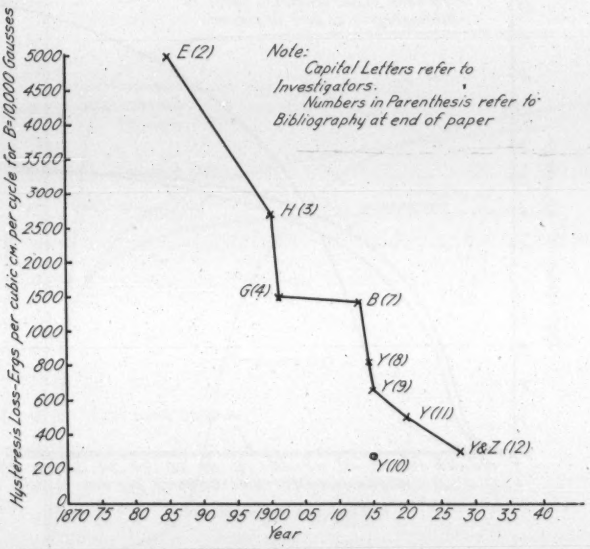


Maximum permeability of iron.

The effect of silicon was attributed to its cleansing action in removing oxides, and gave a hint that even greater values might be expected. By investigating the effect of minute quantities of impurities (C, P, S, Mn) and plotting the effect of carbon after eliminating the effect of the others the curves extrapolated to zero per cent. carbon indicated the possibility of zero hysteresis and enormous permeabilities for a single crystal of the element Fe.[11] Our approach to this goal will of necessity be assymptotic. Recently, by melting electrolytic iron with a small amount of carbon added (0.06 per cent.) in a high frequency vacuum furnace, designed and developed by P. H. Brace and N. Ziegler,[16] a sample was produced which after forging, machining into rings 2.5 × 1.9

× .3 cm. (1″ × ¾″ × ⅛″) and annealing at 900° gave a maximum permeability of 61,000, an initial permeability of 1150, and a hysteresis loss of 300 ergs per cubic cm. per cycle for B = 10,000. This is probably the purest sample of "iron" made to date, but is far from Fe, as it contains a

FIG. 2.



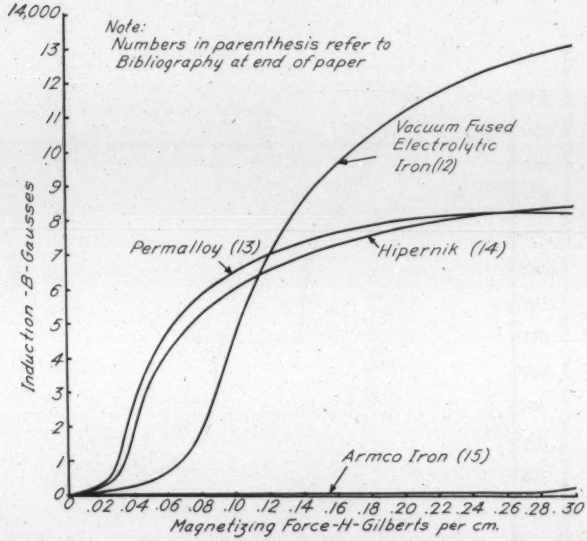
Hysteresis loss of "iron"

few thousandths per cent. carbon, some oxygen, and traces of sulphur, phosphorus, silicon and manganese. Its high magnetic quality may be due to its low oxygen content, obtained by the small addition of carbon and the high vacuum used in melting (0.0001 mm. Hg). So far we have no accurate data as to the oxygen and nitrogen content and their effect on the magnetic properties, but such data are now being collected.

In the above discussion the maximum permeability has

been used as a measure of the improvements; hysteresis loss could equally well have been used, as is seen in Fig. 2, decreasing from 5000 in 1885 to 300 in 1928. In either case the

FIG. 3.



B–H curves for various materials.

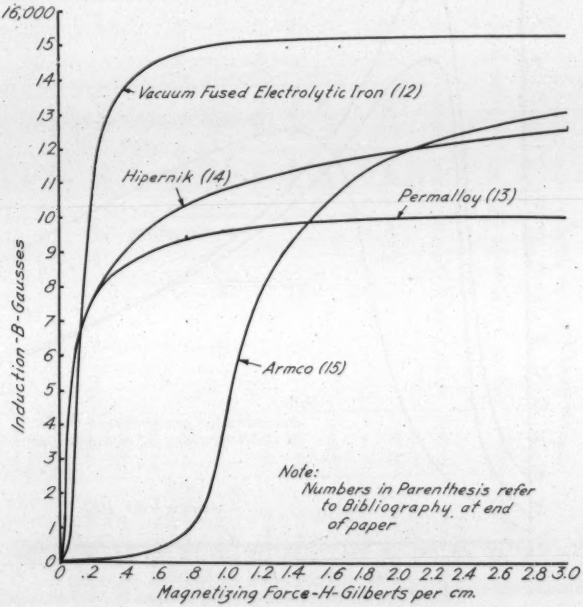
magnetic properties of "iron" have covered a range with a ratio of around 20 to 1 between the first and last representatives: max. permeabilities 2500 to 61,000, hysteresis loss 5000 to 300. Similarly, the initial permeability has been increased from a value less than 250 to 1150, a ratio of 5 to 1.

To use "iron" as a standard for comparison and to state for example that some alloy is 100 times more magnetic than it, is thus obviously meaningless, unless the particular kind of "iron" and the treatment to which it has been subjected are specified.

In Figs. 3, 4 and 5 are given B–H and μ–H curves for

four important materials: Armco iron (the purest commercial grade of iron available), vacuum fused electrolytic iron, Hipernik (50 per cent. Fe-Ni Alloy), and Permalloy (78½ per cent. Fe-Ni Alloy). The curves have all been obtained by the same method of testing (ballistic ring method) and

FIG. 4.



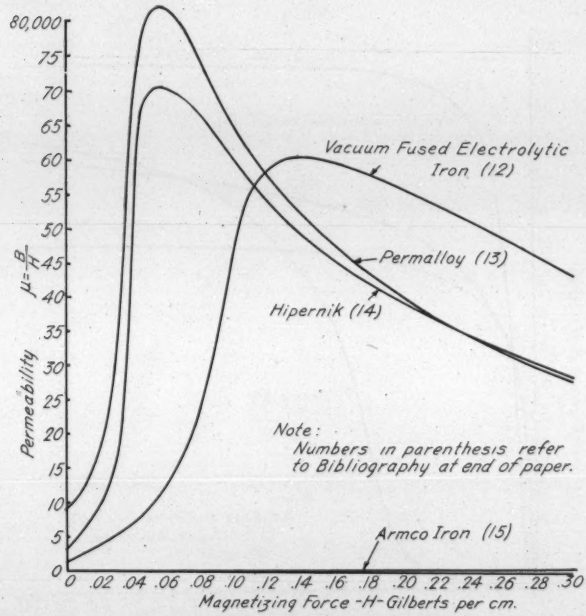
B–H curves for various materials.

are, therefore, directly comparable. The curves for Hipernik and Permalloy are not the best obtained,* but represent properties that are readily obtainable. The curves for vacuum fused electrolytic iron are the best ever obtained for this

* Arnold and Elmen [13] have mentioned max. permeabilities of 100,000 and initial permeabilities of 10,000 for Permalloy, and max. permeabilities above 80,000 have been obtained by the writer for Hipernik.[14]

material, but there is no reason to believe that they are the best obtainable when we know how to reduce the impurities still further, particularly carbon and oxygen, simultaneously. The curves for Armco iron are representative curves.*

FIG. 5.



Magnetic permeability curves for various materials.

In comparing these four materials we see from Fig. 5 that the expression "one hundred times more magnetic than iron" might be justified if we compare Permalloy or Hipernik with Armco or other commercial grades of iron for low values of H, but this expression can with equal justification be used in comparing the vacuum fused electrolytic iron with Armco

* Armco Ingot Iron is used for comparison because of its excellent magnetic properties particularly at high inductions for which the permeability approaches that for vacuum fused electrolytic iron.

Magnetic Properties of "Iron."
1870–1928

Year	Investigator	Material	Max. Permeab. $\mu_{Max.}$	Permeability for H = gilberts per cm.					For B = 10000		Refer.*
				0	.01	.1	.5	1	Coerc. Force Gilberts/cm.	Hyst. Loss Ergs/cc./cycle	
1873	Rowland	Wrought Iron	2,500								1
1885	Ewing	Wrought Iron	2,600							5,000	2
1900	Hadfield	Sw. Char. Iron	2,600				1,000	2,000	.92	2,700	3
1901	Gumlich and Schmidt	Wrought Iron	8,350						.50	Abt. 1,500	4
1910	Terry	Electrol. Iron as dep. and annealed	11,000								5
1912	Gumlich and Goerens	Sheets containing 0.4% Silicon	11,600						Abt. .45	Abt. 1,400	6
1913	Breslauer	Fisher Electrol. Iron as dep. and annealed	11,500				11,200	9,600		1,440	7
1914	Yensen (Univ. of Ill.)	Vacuum fused electrol. iron	19,000				18,800	12,500	Abt. .22	810	8
1915	Yensen (Univ. of Ill.)	Vacuum fused electrol. iron	25,800				23,600	14,000	.20	660	9
1915	Yensen (Univ. of Ill.)	Vacuum fused electrol. iron with .15% silicon added	50,000				27,000	14,500	.09	290	10
1920	Yensen (W. E. & M. Co.)	Vacuum fused electrol. iron	41,500			1,700	27,000	14,600	.17	500	11
1928	Yensen and Ziegler (W. E. & M. Co.)	Vacuum fused electrol. iron	61,000	1,150	2,600	46,600	28,600	15,500	.09	300	12
		For Comparison: Permalloy (78% Ni) and Hipernik (50% Ni) and Armco Iron.									
1923	Arnold and Elmen	Permalloy (78% Ni)	87,000	9,000	12,000	65,000	18,000	9,500	.05±	180±	13
1925	Yensen	Hipernik (50% Ni)	70,000	3,000	6,100	60,000	19,000	10,000	.05	200	14
1920	Arnold and Elmen	Armco Iron	7,000	250	260	320	1,000	4,300	.72	2,100	15
1916	Yensen										

* See Bibliography at end of paper.

iron. For $H = .15$, for example, the permeability of the latter is 260 while that of the former is 60,000, i.e., the ratio is as 1 to 230. In other words, one "iron" is 230 "times more magnetic" than another "iron," and the difference in permeability between these two kinds of iron is greater than that between the best "iron" and Permalloy or Hipernik for any value of H except perhaps for $H = 0$.

As far as the utility of this remarkably good "iron" is concerned, nothing much can be said at the present time because of its great susceptibility to other elements, principally carbon, oxygen, and perhaps nitrogen. Aside from the difficulty of its preparation in the first place, it is very difficult to obtain it in the form of thin sheets without ruining the magnetic properties beyond repair, largely because of oxidation and grain refinement. But it is of great scientific value. The results obtained confirm the prediction as to the magnetic properties of Fe, and thereby help us another step forward on the difficult road to an understanding of the nature of ferro-magnetism. It has been thought that the initial bend of the ordinary B–H curve (as illustrated for Armco iron in Fig. 4, for $H = 0$ to 0.8) is an inherent characteristic of "iron." Now we see that the purer we make the "iron," the less pronounced this initial bend becomes, and it is not inconceivable that it may disappear for the perfect Fe crystal.

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THE EQUATION OF STATE OF A MIXTURE DETERMINED FROM THE EQUATIONS OF STATE OF ITS CONSTITUENTS, AND ITS APPLICATION IN DETERMINING THE PHYSICAL AND CHEMICAL PROPERTIES OF A MIXTURE IN TERMS OF THOSE OF THE CONSTITUENTS.

BY

R. D. KLEEMAN, B.A., D.Sc.

§ 1. THE RELATION OF VOLUME AND MASS IN THE EQUATION OF STATE OF A MIXTURE.

THE equation of state of a mixture in its most general form involving M_a gram atoms of the constituent a, M_b gram atoms of the constituent b, etc., which may combine to form various molecules, is

$$p = \psi(T, v, M_a, M_b, \cdots), \tag{1}$$

where p, v, and T denote respectively the pressure, volume, and absolute temperature T of the mixture. Since the pressure p does not depend on the total masses of the constituents, but on their partial densities, the quantities v, M_a, M_b, $\cdots$ are associated with each other in the form of the ratios M_a/v, M_b/v, $\cdots$, and hence equation (1) should be written

$$p = \psi\left(T, \frac{M_a}{v}, \frac{M_b}{v}, \cdots\right). \tag{2}$$

§ 2. PROPERTIES OF THE EQUATION OF STATE IN RESPECT TO THE TEMPERATURE

If the masses M_a, M_b, $\cdots$ of the constituents are all zero while v remains finite, p is zero, and hence p may not be a function of T alone. The temperature T therefore appears in the equation only in functions of T which are factors of the ratios M_a/v, M_b/v, $\cdots$, or the foregoing equation should be written

$$p = \psi\left(A_1\frac{M_a}{v}, A_2\frac{M_a}{v}, \cdots B_1\frac{M_b}{v}, B_2\frac{M_b}{v}, \cdots\right), \tag{3}$$

where $A_1, A_2, \cdots B_1, B_2, \cdots C_1, C_2, \cdots$ are functions of T. These functions should initially be looked upon as being entirely independent of each other. If relations exist between them, they would be expressed by separate equations whose existence we need not consider at present.

§ 3. THE GROUPING OF THE TERMS OF THE EQUATION OF STATE, AND ITS FINAL FORM.

Suppose that the constituents $b, c, \cdots$ become the same in nature as the constituent a. The functions $B_1, B_2, \cdots C_1, C_2, \cdots$ then become the same in nature as the functions $A_1, A_2, \cdots$. It will not lessen the generality of the investigation if we suppose that under these conditions $B_1 = A_1$, $B_2 = A_2, \cdots C_1 = A_1, C_2 = A_2, \cdots$, etc. Let us now write $M_b = h_b M_a$, $M_c = h_c M_a, \cdots$. We are under these conditions dealing with $(1 + h_b + h_c + \cdots) M_a$ gram atoms of the substance a. The function ψ in equation (3) should therefore possess the property expressed by the equation

$$\psi\left(A_1\frac{M_a}{v}, A_2\frac{M_a}{v}, \cdots A_1h_b\frac{M_a}{v}, A_2h_b\frac{M_a}{v}, \cdots\right) = \psi\left(A_1(1 + h_b + h_c + \cdots)\frac{M_a}{v}, A_2(1 + h_b + h_c + \cdots)\frac{M_a}{v}, \cdots\right). \quad (4)$$

This is realized only if equation (3) has the form

$$p = \psi\left(\frac{A_1M_a + B_1M_b + \cdots}{v}, \frac{A_2M_a + B_2M_b + \cdots}{v}, \cdots\right). \quad (5)$$

as can easily be shown by trial.

From this equation the equations of state of the constituents in the pure state may be obtained by writing the masses of the constituents zero except one, giving

$$\left.\begin{aligned} p_a &= \psi\left(A_1\frac{M_a}{v}, A_2\frac{M_a}{v}, \cdots\right), \\ p_b &= \psi\left(B_1\frac{M_b}{v}, B_2\frac{M_b}{v}, \cdots\right), \\ &\cdots\cdots\cdots\cdots \end{aligned}\right\} \cdots \quad (6)$$

where p_a, p_b, $\cdots$ denote their pressures. These have, it will be noticed, the same functional form, which is therefore independent of the nature of the mixture, or pure substances, we are dealing with.

Hence if the equations of state of a number of pure substances be known, the equation of state of any given mixture of these constituents may immediately be written down.

It should be noticed that the temperature function A_1 need not be the same in form as the temperature function B_1, and A_2 not the same in form as B_2, etc. But the number of functions A_1, A_2, $\cdots$ must be equal to the number of functions B_1, B_2, $\cdots$, etc., since the form of ψ is independent. This number is probably small. Meslin [1] has shown that any equation of state which contains three constants-equal in number to the variables temperature, volume, and pressure, which determine the state of the substance, gives the relation of corresponding states. From what we know of this relation it appears that it holds most exactly for non-dissociated and non-polymerized substances. Since the form of ψ is independent we may determine it from such substances, each of which contains atoms of one kind only. It appears, therefore, that the number of functions A_1, A_2, $\cdots$, etc., is probably not greater than three, and if greater, three of them only are of considerable importance.

The foregoing result may be obtained by another method which is instructive. The atoms of a pure substance a may be divided into two parts of masses M_a' and M_a'', in which case the first of equations (6) may be written

$$p_a = \psi\left(\frac{A_1M_a' + A_1M_a''}{v}, \frac{A_2M_a' + A_2M_a''}{v}, \cdots\right).$$

Now let us suppose that the atoms a of mass M_a'' get their nature changed and become atoms b, whose mass will now be written M_b and that of the remaining atoms written M_a. In the case that $M_a'' = M_b = 0$ the equation should give the equation of state of the pure substance a of mass M_a. Hence the process of the molecules a of mass M_a'' becoming molecules b will not change the factors of the M_a''s in the foregoing equation, nor change the functional form of ψ.

[1] *Compt. rend.*, 116, 135 (1893).

But the factors of the M_a'''s will change and may now be written B_1, B_2, $\cdots$, since when $M_a' = M_a = 0$ the equation should give the equation of state of the pure substance b. Thus finally the equation of state of a mixture of substances a and b has the form

$$p = \psi\left(\frac{A_1M_a + B_1M_b}{v}, \frac{A_2M_a + B_2M_b}{v}, \cdots\right).$$

§ 4. ON THE NUMBER OF FORMS OF ψ.

The number of equations of state expressing one variable as a function of the other variables corresponding to a given mixture of constituents (which may be wholly or partly combined with each other) may be greater than unity. This is definitely shown by the existence of isomers. Moreover, we have good reason to believe that in certain cases the constituents of a mixture may be in perfect equilibrium at a certain temperature without combining with each other, but that when the temperature is raised beyond a certain point combination takes place, and the resultant molecules remain in perfect equilibrium when the temperature is lowered to its initial value. It is difficult to prove this because a mixture of substances may appear to be in perfect equilibrium when actually combination is taking place, though extremely slowly. The various functional forms of ψ in equation (5) most likely correspond to the roots of p when the equation of state is written in a form not involving the extraction of roots.

It is also not improbable that the equation of state may have different forms only over a certain range of volume and temperature. This would mathematically be expressed by

$$p=\psi\left(\frac{A_1M_a+B_1M_b+\cdots}{v}, \frac{A_2M_a+B_2M_b+\cdots}{v}, \cdots Z\right), \quad (7)$$

$$\phi\left(Z, \frac{A_1M_a+B_1M_b+\cdots}{v}, \frac{A_2M_a+B_2M_b+\cdots}{v}, \cdots\right)=0, \quad (8)$$

where the number of real values of Z depends on the values of the variables T, v, M_a, $\cdots$. The equations of state of the constituents would have a corresponding number of different forms.

The subject is manifestly an important one, especially in connection with the present investigation, but about which we know little or nothing. It is therefore desirable that it should be systematically investigated. This might lead to the discovery of the exact forms of the function ψ, which is of fundamental importance, and whose discovery would correspond to a great advance in science. An empirical form is permissible only under certain conditions for obvious mathematical reasons.

§ 5. THE APPLICATION OF AN EMPIRICAL SERIES FORM, AND VAN DER WAALS' FORM OF THE EQUATION OF STATE.

These forms are likely to give fair results when the constituents of a given mixture combine only to a slight extent, or not at all. Thus let us consider a mixture of the two constituents a and e which do not combine with each other, and whose equations of state in the form of series when isolated are

$$p_a = A_1\frac{M_a}{v} + A_2\left(\frac{M_a}{v}\right)^2 + \cdots, \tag{9}$$

$$p_e = E_1\frac{M_e}{v} + E_2\left(\frac{M_e}{v}\right)^2 + \cdots, \tag{10}$$

where A_1, A_2, $\cdots$ E_1, E_2, $\cdots$ are functions of T. The equation of state of the mixture would then be

$$p = \frac{A_1M_a + E_1M_e}{v} + \frac{(\sqrt{A_2}M_a + \sqrt{E_2}M_e)^2}{v^2} + \cdots. \tag{11}$$

It should be pointed out that we cannot write $A_2 = A_2'A_2''$ and $E_2 = E_2'E_2''$ in equations (9) and (10) in order to obtain for the corresponding term in the equation of state of the mixture $(A_2'M_a + E_2'M_e)(A_2''M_a + E_2''M_e)1/v^2$, which is indeterminate since we may give to A_2' and E_2' any value we please. It will be recognized now that in all cases in passing from the equations of state of the constituents to that of the mixture it must be done in such a way that no ambiguity arises.

Fair results should also be obtained with a series form of the equation of state in the case of an interacting mixture, if

we knew the nature and proportion of the various molecules present, and the complex molecules are sufficiently stable that their equation of state when isolated can be obtained. Thus suppose that in a mixture of molecules a and e molecules ae are formed, and that it contains M_a, M_e, and M_{ae} mols of the molecules a, e, and ae respectively. And suppose that the molecules ae in the isolated state decompose so slowly that their equation of state may be obtained, which expressed as a series would be

$$p_{ae} = X_1 \frac{M_{ae}}{v} + X_2 \left(\frac{M_{ae}}{v} \right)^2 + \cdots, \tag{12}$$

where X_1, X_2, $\cdots$ are functions of T. The equation of state of the mixture is then given by

$$p = \frac{A_1 M_a + E_1 M_e + X_1 M_{ae}}{v} + \frac{(\sqrt{A_2} M_a + \sqrt{E_2} M_e + \sqrt{X_2} M_{ae})^2}{v^2} + \cdots. \tag{13}$$

But it should be remembered that if we knew the exact forms of the function ψ the equations of state of such a mixture, ifs in equilibrium, would immediately be given by equation (5).

This method may also be applied to a mixture which is not in perfect equilibrium, but undergoes slow changes in molecular constitution.

If the van der Waals' equations of state of the pure substances a and e are given by

$$p_a = \frac{M_a RT}{v(1 - b_a M_a/v)} - \frac{a_a M_a^2}{v^2}, \tag{14}$$

$$p_e = \frac{M_e RT}{v(1 - b_e M_e/v)} - \frac{a_e M_e^2}{v^2}, \tag{15}$$

where b_a and a_a denote the co-volume and intrinsic pressure constant of the substance a, and b_e and a_e the corresponding quantities of the substance e, the equation of state of the mixture is

$$p = \frac{(M_a + M_e)RT}{v(1 - b_a M_a/v - b_e M_e/v)} - \frac{(\sqrt{a_a} M_a + \sqrt{a_e} M_e)^2}{v^2}. \tag{16}$$

This equation is likely to hold only for true mixtures, to an extent depending on the closeness the facts may be represented by an equation of the van der Waals' type. Applications of the equation will be given in a subsequent paper.

§ 6. FURTHER DIRECT APPLICATIONS OF THE RESULTS OF SECTION 3.

(*a*) The nature of the phases into which a mixture divides may be determined from the continuity equation of state of the mixture, or from the kind of equation we are dealing with in this paper, i.e., equation (5), by a method that will be published in a separate paper. It follows therefore that if the nature of the function ψ were known the nature of the phases of a mixture could be determined from the equations of state of the constituents. When there is little chemical interaction between the constituents this determination may be carried out by means of a series form of the equation of state dealt with in the previous Section.

(*b*) Another important application of equation (5) is the determination of the equation of state of an *interacting gaseous* mixture containing various molecules, from the equations of state of the constituents. But the latter equations correspond approximately to the equation of a perfect gas. It does not therefore seem possible at first sight that an equation which does not obey the gas laws could be deduced from these equations. But that this is mathematically possible will appear from considering the term

$$1/1 - e^{\left(\frac{A_1M_a+E_1M_e}{v}\right)^2},$$

which we will suppose forms part of the equation of state of a mixture of substances a and e. If the expression $A_1M_a + E_1M_e$ is small, in which case A_1 and E_1 have opposite signs, the exponential term is nearly equal to unity, and the whole term therefore large. If we write $M_e = 0$ in the term it becomes

$$1/1 - e^{\left(\frac{A_1M_a}{v}\right)^2}$$

and is then a term in the equation of state of the pure substance a. Now although the expression just considered may be small it does not follow therefore that the term $(A_1M_a/v)^2$

also is small, in fact it may have a large value. In that case the exponential term is large, and the whole term small. Similarly if we write $M_a = 0$ in the term it becomes one of the terms of the equation of state of the pure substance e. The value of its exponential term is large because the difference between $A_1 M_a/v$ and $E_1 M_e/v$ is small, and hence the value of the whole term is small. Thus this term will be of negligible value in the equations of state of the constituents, but not in the equation of state of the mixture.

It will be recognized now how important it is to discover the exact forms of the function ψ, and why empirical forms will not be of use in many cases. The stated problem can therefore at present be solved theoretically only. Its solution is very important because it would enable us to determine the nature of the molecules that may be formed in the gaseous state from given constituents by considering their equations of state in the pure state.

(*c*) Equation (5) may also be used to calculate the specific heat of a mixture by the help of the well known equation

$$\frac{\partial u}{\partial v} = T\frac{\partial p}{\partial T} - p, \tag{17}$$

where u denotes the internal energy of the mixture at the pressure p, volume v, and absolute temperature T. On multiplying this equation by ∂v and integrating it between the limits v and ∞ it may be written

$$u_\infty - u = \int_v^\infty \left\{ T\frac{\partial p}{\partial T} - p \right\} \partial v, \tag{18}$$

where u_∞ denotes the internal energy of the mixture at infinite volume. On differentiating this equation with respect to T at constant volume it becomes

$$c_{v\infty} - c_v = T\int_v^\infty \frac{\partial^2 p}{\partial T^2} \cdot \partial v, \tag{19}$$

where $c_{v\infty}$ and c_v denote the specific heats of the mixture at the constant volumes v and ∞ respectively. For p we may now substitute from the equation of state of the mixture derived from the equations of state of the constituents. It

remains to determine $c_{v\infty}$, which can be shown to be equal to the sum of the specific peats of the isolated constituents.

The expression for the internal energy u must possess the property that if the masses of the constituents are increased n times at constant pressure, in which case the volume is increased n times, the value of u is increased n times. This is realized if the expression has the form

$$u = M_a \cdot \psi_a\left(T, \frac{M_a}{v}, \frac{M_b}{v}, \cdots\right) + M_b \cdot \psi_b\left(T, \frac{M_a}{v}, \frac{M_b}{v}, \cdots\right) + \cdots M_a \cdot \phi_a(T) + M_b \cdot \phi_b(T) + \cdots, \tag{20}$$

or

$$u = \Sigma M \cdot \psi\left(T, \frac{M_a}{v}, \frac{M_b}{v}, \cdots\right) + \Sigma M \cdot \phi(T). \tag{21}$$

The value of the internal energy at infinite volume is obtained by writing $v = \infty$ in the foregoing equations. The value of the internal energy of the isolated constituent a at infinite volume is obtained on writing $M_b = 0$, $M_c = 0$, $\cdots$, and $v = \infty$ in these equations, and so on for the other constituents. Now the value of the function ψ_a is evidently the same in each case, and this also holds for the other functions. The internal energy of the mixture at infinite volume is therefore equal to the sum of the internal energies of the isolated constituents. It immediately follows that this holds also for the specific heats.

An important extension may be made of the foregoing investigation. Suppose that the mixture is subjected to a magnetic field and that u represents the internal energy due to the presence of the field. It can be shown as before that an equation of the form (20) should hold. Similarly as before it follows then that the magnetic energy of the mixture at infinite volume is equal to the sum of the magnetic energies of the isolated constituents. If an electric field is applied instead of a magnetic field it can similarly be shown that the electric energy of the mixture at infinite volume is equal to the sum of the electric energies of the isolated constituents.

§ 7. THE DETERMINATION OF THE PHYSICAL PROPERTIES OF A COMPLEX SUBSTANCE FROM THE CORRESPONDING PROPERTIES OF THE CONSTITUENTS.

It should be possible to express every property of a complex substance in terms of the same properties of the constituents. For evidently there cannot exist any property of the substance the possibility of which did not exist in the molecules or atoms of the constituents. Thus the various magnetic, electric, optical and other properties of—say HCl, should be possible of being expressed in terms of those of H_2 and Cl_2. Now it will appear that if the exact forms of the function ψ were known these determinations could usually be carried out. Examples of the method of procedure, which varies with different cases, will now be given.

(*a*) Let us suppose that a mixture and its isolated constituents are subjected to an electro-magnetic radiation which falls upon them at a certain angle. The radiation will exert pressures on the substances which depend on the reflecting powers of the substances for the particular radiation we are dealing with. Now if the pressures exerted on the constituents were known for various volumes and temperatures, the pressure exerted on the mixture may be calculated. Thus let the equations of state of the isolated constituents *a* and *e* under these conditions be given by

$$p_a' = \psi\left(\frac{A_1'M_a}{v}, \quad \frac{A_2'M_a}{v}, \quad \cdots\right), \tag{22}$$

$$p_e' = \psi\left(\frac{E_1'M_e}{v} \quad \frac{E_2'M_e}{v}, \quad \cdots\right). \tag{23}$$

The functions A_1', A_2', $\cdots$ E_1', E_2', $\cdots$ (which may be given any empirical forms of which the constants are then determined) are determined from values of p_a' corresponding to different volumes and temperatures of the substance, and the values of p_a' are determined from the equation of state of the substance when not subjected to a radiation, and a knowledge of the reflecting powers of the substance under various conditions. Similarly the functions E_1', E_2', $\cdots$ are determined. The equation of state of the mixture is then

given by

$$p' = \psi\left(\frac{A_1'M_a + E_1'M_e}{v}, \quad \frac{A_2'M_a + E_2'M_e}{v}, \quad \cdots\right). \quad (24)$$

Hence the pressure p_r exerted by the radiation on the mixture is

$$p_r = p - p', \quad (25)$$

where p denotes the pressure when no radiation is acting. From this the reflecting and absorbing powers of the mixture under the given conditions may be determined.

At present it is impossible to carry out such calculations through a lack of sufficiently extensive data, and because the exact nature of ψ is not known. But the existence of such relations as indicated may be helpful in predicting various properties of the reflecting power. Thus for example if matter obeyed the gas equation under all conditions it would follow that the reflecting power of a mixture is additive in nature. Since the deviations from the gas laws are due to the existence of the molecular forces of interaction, it follows that the reflecting power of a molecule is affected by the external forces acting upon it.

(*b*) The magnetic permeability of a mixture may be calculated from the permeabilities of the constituents. Thus let the equation of state of the isolated substances a and e in a magnetic field of intensity H be given by equations (22) and (23), where the functions A_1', A_2', $\cdots$ E_1', E_2', $\cdots$ refer to the new conditions. The function ψ remains the same in form since it is independent of the nature of substances, and the application of a magnetic field to a substance might only change somewhat its nature. The functions A_1', A_2', $\cdots$ E_1', E_2', $\cdots$ are determined by the help of the equations

$$\frac{\partial u_a'}{\partial v} = T\frac{\partial p_a'}{\partial T} - p_a', \quad (26)$$

$$\frac{\partial u_e'}{\partial v} = T\frac{\partial p_e'}{\partial T} - p_e', \quad (27)$$

where u_a' and u_e' denote the internal energies at the volume v of the isolated substances a and e when under the action of

the magnetic field. Now

$$u_a' = u_a + \mu_a H^2 v/8\pi, \tag{28}$$

$$u_e' = u_e + \mu_e H^2 v/8\pi, \tag{29}$$

where u_a and u_e denote the internal energies when there is no field acting, and μ_a and μ_e the permeabilities of the substances a and e respectively. Equations (26) and (27) may now be written

$$T\frac{\partial p_a}{\partial T} - p_a + \frac{H^2}{8\pi}\frac{\partial(\mu_a v)}{\partial v} = T\frac{\partial p_a'}{\partial T} - p_a', \tag{30}$$

$$T\frac{\partial p_e}{\partial T} - p_e + \frac{H^2}{8\pi}\frac{\partial(\mu_e v)}{\partial v} = T\frac{\partial p_e'}{\partial T} - p_e', \tag{31}$$

since

$$\frac{\partial u_a}{\partial v} = T\frac{\partial p_a}{\partial T} - p_a, \tag{32}$$

$$\frac{\partial u_e}{\partial v} = T\frac{\partial p_e}{\partial T} - p_e, \tag{33}$$

where p_a and p_e are the pressures of the substances a and e in the absence of the field. On substituting for p_a' and p_e' from equations (22) and (23) in equations (30) and (31) they may be used to determine the functions A_1', A_2', $\cdots$ E_1', E_2', $\cdots$ from values of p_a, p_e, μ_a, and μ_e corresponding to various values of v and T. The equation of state of the mixture under the field H is then given by equation (24). The permeability μ of the mixture is obtained by the help of the equation

$$T\frac{\partial p}{\partial T} - p + \frac{H^2}{8\pi}\frac{\partial(\mu v)}{\partial v} = T\frac{\partial p'}{\partial T} - p' \tag{34}$$

obtained similarly as equations (30) and (31). On multiplying the equation by ∂v and integrating it between the limits v and ∞ it becomes

$$\mu_\infty \cdot \infty - \mu v = \frac{8\pi}{H^2}\int_v^\infty \left\{ T\frac{\partial(p' - p)}{\partial T} - (p' - p) \right\} \partial v, \tag{35}$$

where μ_∞ denotes the permeability at infinite volume. The value of μ_∞ has been shown in Section 6 to be equal to the sum

of the permeabilities of the isolated constituents, and if p' and p are substituted from equations (24) and (5) the permeability μ may be calculated.

In a similar manner the inductivity of a mixture may be calculated from the inductivities of the constituents.

Every property of matter with which a force may be associated can be dealt with along similar lines. Thus for example suppose that a substance is placed in a liquid medium of a given electrical conductivity through which an electrical current is flowing. If the electric conductivity of the substance is not the same as that of the medium certain forces will act upon it. These may be made the basis for calculating the electric conductivity of a mixture in terms of the conductivities of the constituents. Similarly the heat conductivity may be investigated.

The foregoing is only an outline of the uses to which equation (5) may be put; the subject will be further developed in a subsequent paper.

Preliminary Experiments in Obtaining Extremely High Electrical Potentials from Atmospheric Electricity. A. BRASCH, F. LANGE AND C. URBAN. (*Naturwissenschaften*, Feb. 17, '28.) Were it possible to get very high potentials they could be used to produce streams of particles with great velocities that would further the study of atomic disintegration. On account of the technical difficulties in the way of attaining high voltages by using transformers or other electrical apparatus the authors turned to atmospheric electricity following the good example of Benjamin Franklin in his famous Philadelphia experiment. Their experiments were conducted on Monte Generoso near Lugano in the summer of 1927. This mountain was selected because of its accessibility and of the frequency of its thunderstorms.

"The normal electrical field in the atmosphere of the earth, which generally amounts to about 200 volts per meter, is raised during thunderstorms to some hundreds of kilovolts per m. A point only 100 m. above the ground has accordingly a potential from five to thirty million volts above that of the earth. To avail themselves of this the authors swung over a valley a line carrying several hundred square meters of coarse wire gauze provided with points. The line was 760 m. long and the gauze was 80 m. from the ground. The line was protected at each end by a series of insulators. To prevent the formation of the corona with consequent loss of energy thin, round, metallic bodies were strung like beads on the half of the line extending from the gauze to the measuring instruments.

The installation was not ready for operation until the end of August by which time the season for storms had about passed. Fortunately one thunderstorm did, however come. At the height of the storm sparks jumped across a gap 4.5 m. long from a body connected to the suspension line and collecting gauze to an earthed point. The observers believe that the gap could have been greatly lengthened without causing the sparks to cease. They estimate that 1,700,000 volts were obtained by their device. By suspending the gauze at a greater elevation higher potentials can be got. These experiments will be continued in the summer of 1928.

G. F. S.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

INTERNATIONAL ASTRONOMICAL UNION.

THE third general assembly of the International Astronomical Union took place in Leiden, Holland, during the week of July 5 to 13. It was probably the largest gathering of astronomers in history; representatives from 28 different countries were present.

The main work of the Union was performed in sessions of its 28 commissions, each concerned with some particular phase of astronomical or astrophysical science. The Bureau of Standards was represented on the commission for standards of wave-length and tables of solar spectra. The spectroscopy section of this bureau was the first to determine new standards of wave-length both in laboratory arc spectra and in the solar spectrum. These new determinations were directly responsible for the revision and adoption of two sets of international standards of wave-length, one set derived from the iron arc and the other from the sun. Both sets are based on the primary cadmium or equivalent neon standard and the individual values are now believed to be correct to 1 part in 4 or 5 millions. Each system of standards covers approximately an octave of the spectrum, ultra-violet to red. The system of laboratory standards consists of wave-lengths corresponding to about 300 bright lines in the emission spectrum of the iron arc and the solar standards are represented by wave-lengths corresponding to about 400 dark lines in the absorption spectrum of the sun. The iron standards replace a similar system which has been in international use for about a decade but which was recently found to contain systematic errors of the order of magnitude of 1 part in 1 million. The solar standards will replace the values determined by Professor Rowland about 40 years ago.

In addition to adopting these revised standards the International Astronomical Union advised the investigation of vacuum arc and furnace spectra to determine if their use

* Communicated by the Director.

will permit further improvement in laboratory standards of wave-length and recommended the extension of the tables of standards to longer and to shorter waves, placing special emphasis on the importance of better standards among the very short waves.

SCIENTIFIC BY-PRODUCT OF LEVULOSE INVESTIGATION.

THE research on the identification of the unknown constituents of hydrolyzed inulin has resulted in the isolation and description of one well defined crystalline substance.

Inulin, the starch-like substance found in the juice of dahlia tubers, may be converted by hydrolysis or treatment with acid into the sugar known as levulose or fructose. It was discovered that but 92 per cent of the expected amount of fructose was obtained by this hydrolysis. Since it seems probable that fructose will take its place as a large-scale commodity it was important to learn the nature of so large an impurity in hydrolyzed inulin. By removing the fructose by precipitation with lime a residual sirup was obtained which instead of rotating polarized light 91 degrees to the left as does fructose, rotated 55 degrees to the right. This sirup proved to be a mixture of sugars none of which could be isolated.

By treatment with very strong acid at boiling temperature a partial hydrolysis was effected and it was found that the sole product of hydrolysis was fructose.

The mixture of sugars was then converted to their acetates and from the mixture one acetylated sugar crystallized and was purified and analyzed. This was then converted back to the original sugar which was finally obtained in crystalline form. This sugar, which has never hitherto been isolated, proved to be a disaccharide composed of two molecules of fructose combined to form a compound sugar rotating polarized light to the right. It is conjectured that it is an integral part of the inulin molecule and will ultimately assist in the determination of the structure of inulin.

LINEAR THERMAL EXPANSION OF AMBER.

THE linear thermal expansion of a sample of amber purchased from the National Importing Company, Inc., of

New York City, was investigated by this bureau, over various temperature ranges between 0 and 90° C. The results obtained are given in the following table:

Temperature range ° C.	Average coefficient of expansion per degree Centigrade.
0 to 15	0.000047
0 to 30	0.000050
0 to 50	.000053
0 to 90	.000061
15 to 30	.000053
30 to 50	.000058
50 to 90	.000072

Between 0 and 50° C. the coefficient of expansion of this sample of amber is about five times the coefficient of ordinary steel.

HEALTH HAZARDS IN CHROMIUM PLATING.

A PAPER on health hazards in chromium plating, by J. J. Bloomfield, of the Public Health Service, and W. Blum, of the Bureau of Standards, was presented at the meeting of the American Chemical Society at Evanston, Ill., on August 9. This paper has now been published in "Public Health Reports" for September 7, 1928, and reprints may be obtained from either the U. S. Public Health Service or the Bureau of Standards, Washington, D. C. A brief resume of this paper is as follows:

During the past few years chromium plating has developed rapidly, and it is now being extensively applied upon automobiles and plumbing fixtures because of its high luster and resistance to tarnish; and upon printing plates, gages, tools and dies on account of its extreme hardness. The bath used for chromium plating consists principally of chromic acid. During the plating process considerable hydrogen and oxygen are liberated, and these carry a spray of chromic acid into the air.

It has long been known that in the manufacture of chromic acid and chromates, the operators are subject to attack and perforation of the nasal septum; and to the formation of ulcers or "chrome holes" upon the hands or other exposed parts of the body. Accordingly some provisions have been made in all chromium plating plants for artificial ventilation

and for other sanitary measures. In spite of these precautions, however, the employees in some plants have been affected in the above mentioned ways.

In order therefore to determine the extent of the hazard and the best means of overcoming it, a survey was conducted in several commercial plants. This included a study of the methods and degree of ventilation of the concentrations of chromic acid in the air breathed by the workers, and a physical examination of the latter. Although only six plants were visited, and twenty-three persons examined, the results were so consistent in different plants, and agreed so well with previous experience with chromates, that certain tentative conclusions and recommendations are warranted.

It was found that exposure to very low concentrations of chromic acid, e.g., one milligram in 10 cubic meters, or one-sixtieth of a grain in 350 cu. ft. (which is about the volume of air breathed by a worker in eight hours), is sufficient to cause nose bleed and nasal inflammation in a week or less. Higher concentrations or longer exposures cause extensive attack and even complete perforation of the nasal septum. This is painless, however, and the operator may be entirely unaware of the perforation. Many of the employees were found to have chromium ulcers on the hands or other exposed parts of the body. No evidence was found of injury to the respiratory tract except in the nose, nor of any effect upon the digestive system or the kidneys.

While therefore there is a real hazard in chromium plating, it is not critical, and can be entirely eliminated by suitable measures. These should include an effective system of ventilation in which the air is drawn horizontally across the plating tanks into a narrow duct in which the air velocity should be about 2,000 feet per minute. So far as possible rubber gloves, aprons and shoes should be worn. Frequent applications of vaseline or mentholatum salve to the nose and hands greatly reduce the danger or ulceration. All cuts and abrasions of the skin should receive regular inspection and medical treatment.

If these simple, entirely practicable measures are taken, the hazard can be practically eliminated. There is no reason therefore to fear any serious injuries from the extension of chromium plating that is likely to occur in the next few years.

WORKABILITY OF PORTLAND CEMENT PASTES.

Three methods of tests for workability of cement pastes—the extrusion cylinder, the ball plasticimeter, and the capillary tube—were discussed in Technical News Bulletin No. 132 (April, 1928). A fourth method, using the MacMichael viscometer with its modified "paddle" has been studied. This instrument was too small to permit the study of dry mortars. However, a summary of the results already obtained develops two items of interest. First, a comparison was made between the workability of the cements, when tested as neat pastes, as shown by the MacMichael instrument and also as shown by the three methods of test referred to above, which were also reported in the paper published in the *Proceedings of the American Concrete Institute*, "Cement As A Factor In The Workability Of Concrete," Vol. XXIV, 1928, pp. 43–55. There was quite good agreement, in this respect, between the MacMichael instrument and the capillary tube reported in the earlier paper. Secondly, at least within the scope of the instrument used, the relative results of a number of cements tested as neat cement pastes, or in a mortar of a certain proportion, may not be the same as the relative results obtained when testing the same cements in a mortar of different proportions.

TENSILE STRENGTH OF MORTAR IN BRICKWORK.

A series of tests of the adhesion of mortar to sand-lime brick has furnished an opportunity for comparing the tensile strength of the mortar in the brick with the strength of the same mortar in the form of standard briquets. The results herein reported were obtained with a 1 : 1 : 6 cement-lime mortar, in which various amounts of diatomaceous earth had been substituted for equal amounts of lime. The mortars were mixed to a consistency which gave a ½ inch slump in a 2 x 4 inch cylinder. From each batch of mortar, six standard tensile briquets were cast and ten pairs of brick joined together.

The bricks and briquets were aged together in the air of the laboratory for 28 days, after which the tensile strength of the mortar and the adhesion of the mortar to the brick

were determined. The mortar was then removed from the brick to which it adhered, separating readily in slabs about 0.45 inch in thickness. The slabs were cut into specimens about 3 x 1½ inches, care being taken to obtain specimens both from the interior and the edge of the slab.

Saw cuts, ¼ inch deep, were made at the midpoint on each side of the specimens cut from the interior of the slabs, to form a reduced section, one inch across, for the tensile tests. With the specimens cut from the edge of the slab, a single cut only was made from the side opposite the exposed edge, so that this edge might be included in the reduced section being tested.

Strips of light wood placed over two layers of blotting paper were clamped to each end of the specimen. These formed grips by means of which the upper end of the specimen could be attached to a support, and a container for shot attached to the lower end. Shot was poured slowly into the container until the specimen failed. The container was then weighed, and by calculation, the tensile strength of the mortar in pounds per square inch was obtained.

The results are given in the accompanying table. The nature of the slabs from which the specimens were cut was such that equal numbers of specimens could not be obtained from each mortar. In the table, tensile strengths are the average of the number of tests given in the adjoining column except those of the briquets where the figures in all cases were the average of six tests.

It will be noted that in ten of the eleven mixes tested the specimens from the edge of the mortar as it lay in the joint were stronger than those obtained from the interior portion of the joint. Also in eight of the eleven mixes the mortar specimens, both from the edge and the interior, were stronger than the same mortar in briquet form. There are several factors which may affect the strength of the mortar in masonry so that its strength is not necessarily related to that in briquet form. Tapping and jarring as the brick are laid, pointing up the edges, and the absorption of the brick all tend to produce a denser mortar. The interior of mortar in brickwork may be kept in a damp condition longer than the mortar in briquet form, thus tending to increase the strength of a

cement mortar. The surface exposure is relatively less in the masonry joint than in the briquet, and consequently carbonation from the air is less. In the particular mortars tested the factors tending to increase the strength were apparently more effective than the others. Whether this would be true of all mortars cannot be predicted from the available data.

TENSILE STRENGTH OF MORTAR IN BRICK.

Diatomaceous Earth Substituted for Lime.	Specimens from Edge.		Specimens from Interior.		Tensile Strength of Mortar in Briquets (Average of 6 Tests).
	Number of Specimens Tested.	Tensile Strength, Average.	Number of Specimens Tested.	Tensile Strength, Average.	
Per Cent.		Lbs./In.^{2}		Lbs./In.^{2}	Lbs./In.^{2}
0	6	41	3	45	53
10	8	53	5	46	59
20	7	69	5	61	51
30	8	56	5	34	53
40	9	75	8	65	59
50	9	73	7	53	43
60	6	82	13	58	54
70	14	70	10	52	40
80	14	80	10	69	41
90	15	73	9	67	27
100	15	65	10	52	36

SAGGER INVESTIGATION.

TECHNICAL News Bulletin No. 130 (February, 1928) gave a summary of the method of procedure being followed for developing sagger bodies according to fundamental properties.

The thermal expansion, moduli of elasticity and rupture at room and elevated temperatures have since been determined with a view of establishing the value of the "resistance to thermal shock factor R" (described in Technical News Bulletin No. 120, April, 1927), which would give an indication of the probable life of a particular sagger body. At this writing the data, obtained on the fired sagger bodies, show that the resistance factor "R" gives a fairly good indication as to the life of the sagger, when subjected to repeated thermal shock, if referred to a group of bodies containing only fine grog or a group containing only coarse grog. It does not appear to apply when comparing the life of a coarse-grogged sagger with that of a fine-grogged sagger. This would indicate that

the stresses set up in the fine-grogged body are of a different type than those in the coarse-grogged body.

It was also found that (1) saggers containing only a porous grog showed greater resistance to thermal shock than those containing a partially vitrified grog; (2) the rate of thermal expansion of the fine bodies was higher than that of the coarse bodies in 67 per cent. of the observations, similar in 20 per cent., and lower in 13 per cent.; (3) a comparison of the thermal expansion of the sagger bodies with that of the two clays composing each of the bodies shows that in 12 out of the 15 instances the combination was advantageously made with respect to either one or both clays.

Sagger bodies similar to the above have been prepared, but have been fired approximately 50° C. higher. These will be tested and the results compared with those obtained on bodies burned at the lower temperature.

RUBBER BINDERS FOR FOUNDRY CORE.

SAND cores used in the foundry industry have played a very important part in the development of the art of molding and in the reduction of molding costs. There still remain, however, some phases of present day foundry core practice that may be improved upon.

Cores as generally used in the foundry, at the present time, owe their strength or coherence to their content of artificial bonding material and to the baking treatment which is given the core. Strength, however, is required only before casting and easy disintegration of the sand core is very desirable after the cast metal has solidified around it.

The removal of cores from the casting is an expensive, laborious and dusty task, often requiring the use of a pneumatic chisel or other tools. Cracked castings, due to hard cores, which do not crush as the metal solidifies and core blows, due to low permeability of the cores are sources of loss to foundries.

Recently there has been developed, at the bureau, a new foundry core sand binder, rubber or some allied material being used as the basis of the core binder. The outstanding advantages are:

1. The cores crush readily, falling to loose sand of their own accord so that the core sand may be poured from the casting instead of having to be dug out.

2. The cores have greater strength than a green sand core and extend the range of jobs to which a readily crushed core may be applied.

3. The cores are not oven baked; they are merely air-dried.

4. The cores are of high permeability and show remarkable freedom from blowing.

The new rubber core binder consists essentially of a solution of unvulcanized rubber in gasoline. The amount and type of rubber binder used in making cores depend upon the type of core sand, the size of core, and the metal to be cast around it. A core-strength equivalent to that of a green sand core or baked oil sand cores can be attained by the use of the rubber binders. The new binders have proved to be successful in producing sand cores for castings of lead, tin, zinc, brass, phos-bronze, bronze, aluminum, iron and steel.

These new rubber types of binders are discussed in considerable detail in our Letter Circular No. 252, Rubber Binders for Foundry Cores, which is now ready for distribution. Address inquiries only to the Bureau of Standards.

RAYON PROJECT.

THE work on rayon at the bureau has centered chiefly on the study of the effect of moisture on the strength and elongation properties. Actual work along these lines was delayed pending the development of adequate testing procedure and the design and standardization of the necessary equipment.

The method for obtaining stress-strain curves, using a multiple-strand specimen, has been refined considerably. In developing this method yarns of other textile fibers, besides rayon, were studied for comparison. The data obtained are being collated for publication.

This multiple strand strength method consists essentially of winding the yarn under controlled uniform tension onto a specimen holder which in turn may be placed in the jaws of the testing machine without further handling.

It has been demonstrated that the method is simple in

operation, gives accurate and reproducible results, and is sufficiently sensitive for the purposes of the main study.

Using this method, comparative results are being obtained on all four types; viscose, cuprammonium, nitrocellulose, and cellulose acetate. Different relative humidities are being used which will indicate what effect the moisture has on strength and stretch.

For the wet tests a device has been designed and built which consists of a tank built around the lower jaw of a Scott tester. This tank is equipped with heaters and temperature control. This equipment permits the specimen to be immersed in water for definite periods of time and under controlled temperature. It is then broken without further manipulation. It has the advantage that the yarn is not handled from the time it is inserted in the jaw until the test is completed.

Some supplementary work is being done on the cellulose acetate type of rayon to determine what takes place when the rayon is boiled in water. It is recognized that a delustering action results and that the properties of the rayon are effected in various ways. The results have indicated that little, if any, regeneration of the cellulose acetate to cellulose takes place as has been advocated in some theories.

Some preliminary work was done to determine the effect of high pressure and of cathode ray exposures on rayon. This showed little promise and was not continued.

NOTES FROM RESEARCH LABORATORY, INCANDESCENT LAMP DEPARTMENT OF THE GENERAL ELECTRIC COMPANY.*

TEMPERATURE AND BRIGHTNESS OF THE NEW STANDARD TYPE OF INCANDESCENT LAMPS.

By W. E. Forsythe and E. M. Watson.

[ABSTRACT.]

THE new standard type of Mazda lamp, which has as its most distinctive feature an inside-frosted bulb, is the result of a successful endeavor to improve the construction of incandescent lamps over the range of ratings that is suitable for domestic lighting and services of like caliber. To satisfy all the demands of good lighting it was found necessary to have six different sized lamps in this new type, that is, 15 watt, 25 watt, 40 watt, 50 watt, 60 watt, and 100 watt.

A number of improvements were made on the lamps during the process of simplification, so that it was possible to make the new type markedly better than the lamps they were intended to replace. The improvement that gave the new type of lamps their distinctive appearance was the method of frosting the bulb on the inside.

Among the other improvements may be mentioned a reduction of bulb size and the coiling of the filament. This last improvement made it possible to reduce the number of supports and still keep the filament in place.

A lamp bulb frosted on the outside by either the sand blasting or the acid etching process absorbs between five to ten per cent. of the light. A 5 per cent. loss of light does not seem high, but it is too high when an effort is being made to produce the best possible light source consistent with practical operation. Frosting the bulbs by the new process on the inside was found to increase their absorption by less than 1 per cent. above that of clear bulbs of the same size and shape.

* Communicated by the Director

The main reason, of course, for frosting the bulb of the lamp is to reduce the brightness of the source, because the tungsten filament in the lamp is operated at such a very high temperature that it makes a very bright source if viewed directly. The brightest point on a number of these inside frosted lamps has been measured, and the results are given in Table I, where the brightnesses of some other standard sources are included for purposes of comparison. In this table also is given the brightness of the filament of the lamp as it would be observed if it were mounted in a clear bulb. It will be noted that the density of frosting adopted by the engineers is such as to reduce the maximum brightness observed by about a factor of fifty. Quite a number of filaments, such as are used in these new lamps, of different wattages were mounted in clear bulbs and their temperatures measured. These values together with the efficiencies of the lamps are also given in Table I.

Brightness of Filaments and Bulbs of Standard Line Lamps and Some Other Sources for Comparison.

Lamp.	Volts.	Watts.	Type.	L/W.	Maximum Temp. °K.	Color Temp. °K.	Maximum Bulb Brightness cand./cm.²	Filament Bright. c./cm.²
Kerosene flame flat wick						1920		1.2
4-watt per candle carbon lamp						2080		55.0
40-watt tungsten straight filament			vacuum			2500		206.0
40-watt tungsten (frosted bulb)			vacuum				2.5	
Sun as observed at earth's surface							165,000.0	
Clear sky, average								0.4
Standard type lamp	115	15	vacuum	8.6	2475		4.0	214.0
" " "	115	25	"	9.5	2510		4.5	245.0
" " "	115	40	"	10.2	2545		5.5	278.0
" " "	115	50	gas-filled	10.5	2660		9.0	430.0
" " "	115	60	"	11.3	2695		9.5	490.0
" " "	115	100	"	13.4	2755		12.5	600.0

VISIBLE RADIATION CHARACTERISTICS OF INCANDESCENT OXIDES.

By Marcella Lindeman Phillips.

Investigation of the visible radiation of certain oxides, with reference to their possible use as illuminants, has extended over a period of more than a century, but few quantitative results have been published.

This work is an investigation of the visible radiation characteristics of various oxides and oxide mixtures when heated by cathode ray bombardment, gas-air and oxy-gas flames, to brightness temperatures ($\lambda = 0.665\ \mu$) from about 1400 to 2000° K. Ordinary gas-air burners, equipped with valves regulating the gas pressures, were used in flame heating. The oxides were kept in the oxidizing part of the flame. A cold cathode discharge tube, connected through a long capillary to an oxygen tank, was used in studying the oxides under cathode ray bombardment. The oxide, mounted in a nickel button, served as the anode, and was viewed through a window at the end of a long arm of the tube. With the window thus at some distance from the hot circle the error due to blackening of the tube during operation was greatly reduced.

The oxides investigated were urania, ceria, lanthana, neodymia, erbia, yttria, zirconia, thoria, alumina, beryllia, magnesia, and mixtures of thoria with one per cent. ceria (the Welsbach mantle mixture), one per cent. and less of urania, one per cent. neodymia, and one per cent. manganese oxide.

An optical pyrometric method of observation was used, measurements of the brightness of the oxide under test being made with a disappearing filament type of pyrometer, equipped successively with red (effective $\lambda = 0.665\ \mu$) and blue (effective $\lambda = 0.467\ \mu$) screens, from which the red brightness temperature and the ratio of red to blue intensity ratio were determined, and with a combination of red and green screens having an effective wave-length approximately equal to the Crova wave-length, from which the candles emitted per square centimeter of surface were determined.

The oxides used, unless otherwise stated, were in as pure a state as could be obtained, and were either pressed as compactly as possible in a hydraulic press, or fused to insure a

smooth surface. The samples of the fused oxides were prepared by melting them in an atomic hydrogen arc by C. W. Hewlett of the General Electric Company Laboratory in Schenectady. Due to the fragility of the pressed oxides, fused oxides were used when possible.

In general, linear relations were found between the logarithm of the red-blue intensity ratio and the reciprocal of the brightness temperature, and between the logarithm of the candles emitted per unit surface area and the logarithm of the brightness temperature. Different modes of heating gave different radiation curves for the same oxide.

NOTES FROM THE RESEARCH LABORATORY, EASTMAN KODAK COMPANY.*

McBAIN-BAKR BALANCE FOR SORPTION OF VAPORS BY FIBROUS AND FILM MATERIALS.[1]

By P. T. Newsome.

A SHORT description with a diagram of an apparatus for the rapid determination of the sorption of vapors by fibrous and film materials. The essential feature is the use of a McBain-Bakr quartz fiber spring balance.

THE DEPENDENCE OF THE RESOLVING POWER OF A PHOTOGRAPHIC MATERIAL ON THE WAVE-LENGTH OF LIGHT.[2]

By O. Sandvik and G. Silberstein.

THE resolving powers of six various emulsions have been investigated. The spectral region covered extends from wave-length 380 $m\mu$ towards the red end of the spectrum as far as the sensitivity of the particular emulsion would permit. The resolving power has a maximum value at a wave-length of 380 $m\mu$, decreasing rapidly in the blue-green to a minimum value at about a wave-length 535 $m\mu$.

It is interesting to note that the change in resolving power with the wave-length corresponds approximately with the change in the absorption coefficient with the wave-length in the spectral region investigated.

* Communicated by the Director.

[1] Communication No. 343 from the Kodak Research Laboratories and published in *Ind. Eng. Chem.* **20**: *827. 1928; Sci. Ind. Phot.* **8A**: *95. 1928.*

[2] Communication No. 344 from the Kodak Research Laboratories and published in *J. Opt. Soc. Amer.* **17**: *107. 1928.*

The Rehabilitation of the Mines in Northern France. L. LAHOUSSAY. (*La Nature*, May 15, '28.) France in 1913 used 65,000,000 tons of fuel of which 24,000,000 were imported. The two departments of Nord and Pas de Calais produced two-thirds of all the coal mined in France and employed 130,000 workers for that purpose. Unfortunately these departments were soon invaded. Even as early as August, 1914, the Anzin mines were lost and by the end of the year two-thirds of this northern coal region was in the hands of the Germans. The lines of battle swayed back and forth over the coal fields of the value of which both sides were fully aware. The Germans had no desire to leave the mines in workable condition and no later than 1915 blew up the casings of pits at Lens and Liévin. Additional destruction was accomplished in 1917. A year later they took up the task systematically but had not completed it before they were driven out. The usual procedure was to attach a charge of explosive to the end of a beam just about equal in length to the diameter of the shaft. The beam was lowered and manipulated so that the charge was held between one end of it and the casing. After the explosion had pierced the casing water poured in from adjacent strata and drowned the mine. Of 212 mines in the possession of the Germans 140 were blown up, though not always with the results desired.

In April, 1919, a survey of the situation showed that 92,600,000 cu. m., or that number of tons, of water had to be removed from the mines. The greater part of this water was situated at depths ranging from 200 to 400 meters, though some of it was 700 or 800 m. below the surface of the earth. The pumping of this water from intercommunicating mines and its discharge into canals in a country as flat as Flanders presented many problems for solution. Different methods of pumping were used for different levels, and the work of restoring the shafts was undertaken with full recognition of the attendant difficulties. To furnish electric current for the operations it became necessary to erect a central station of 18,000 kw. capacity.

The removal of the water began in 1920 and was at its height in 1921. The mines when finally freed from water had been unworked for periods varying from 5 to 12 years. Their timbers were rotten and their metal parts corroded. In spite of all these drawbacks the production of coal was soon resumed and each succeeding year sees it increasing. In 1919 only one per cent. of the pre-war quantity was mined but by 1920 it had become 4 per cent. and in 1927 110 per cent. To any one who saw the mines shortly after the war their rehabilitation in so brief a time must appear as a triumph of the engineers of France. G. F. S.

NOTES FROM THE U. S. BUREAU OF CHEMISTRY AND SOILS.*

AVOCADO OIL.[1]

By G. S. Jamieson, W. F. Baughman, and R. M. Hann.

[ABSTRACT.]

THE fatty oil contained in the pulp of the avocado was investigated. The chemical and physical characteristics were determined. The oil gave an iodine number of 94.4 and a saponification value of 192.6, and the composition was found to be as follows:

Glycerides of:	Per cent.
Oleic acid	77.3
Linolic acid	10.8
Myristic acid	trace
Palmitic acid	6.9
Stearic acid	0.6
Arachidic acid	trace
Unsaponifiable matter	1.6

Experiments showed that the oil could probably be used for the manufacture of hard soap. When expressed from sound fruit that has been partly dehydrated, the oil could be used locally as an edible oil or a cooking oil.

THE CHEMISTRY OF LIGNIN.

II. FRACTIONAL EXTRACTION OF LIGNIN FROM CORN COBS.[1]

By Max Phillips.

[ABSTRACT.]

LIGNIN was fractionally extracted from corn cobs by a 2 per cent. alcoholic sodium hydroxide solution at room temperature, by 2 per cent. aqueous sodium hydroxide at 100° and 135°, and finally by 4 per cent. aqueous sodium hydroxide

* Communicated by the Chief of the Bureau.

[1] Published in *J. of the Oil and Fat Industries*, 5: 202–206, July, 1928.

[1] Published in *J. of American Chemical Society*, 50: 1986–1989, July, 1928.

at 180°. Each method of extraction was continued until no further lignin was obtained, before the next method in the series was employed. The results justify the conclusion that the lignin in corn cobs is unequally combined with the carbohydrates, part of it being loosely bound, possibly in the form of an ester, and the remainder being more firmly held, probably in the form of an ether-like combination.

NOTES FROM UNITED STATES BUREAU OF MINES.*

RECOVERY OF FINE GOLD BY AMALGAMATION.

THE frequency of cases of poor recovery of fine gold by the amalgamation process has led the United States Bureau of Mines, Department of Commerce, to publish an information circular on the subject.

Experience has shown that in many cases error has been made because the true gold content of a particular sample or deposit was not known. Fire assays of representative samples give accurate results and should be considered final in determining the gold content.

The best method for recovering gold depends on the form of its occurrence in the material to be handled. An experienced operator can obtain a good idea of the amount of free gold and can tell something as to the fineness of it by careful panning. The sulphides should be separated and cleaned from the free gold and gangue, and then weighed and assayed. If the sulphide carries gold, it is probable that part of the gold in the slimed portion is locked up with sulphides and will not amalgamate.

It is advisable, if practicable, to have an expert make a microscopic examination and report on minerals present and how associated in the gangue. Such an examination is needed by the examining mining engineer or metallurgist before he can recommend methods of recovery.

The fine gold lost in the usual amalgamation process is often called "float gold." Most of this gold is probably in the form of thin laminæ, flattened grains, or scales, and its loss is due to noncontact with the mercury. A thinner pulp, provided by the introduction of swinging amalgamated plates as obstructions in the pulp flow and the stirring action caused by more frequent drops onto the different sections of the amalgamation plates has improved the recovery of such gold. Fine gold is readily recoverable by good contact with the

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amalgamation plates and should not be confused with the gold included in fine sulphides or in other nonamalgamating forms.

"Rusty gold" is a term that has been adopted to designate gold which, though apparently free, does not readily amalgamate. It is known that a thin film of sulphur, oxide of iron, silica, grease or other substances may cover the surface of the gold particle and prevent amalgamation. The film is usually attacked by grinding or some form of abrasion and by the use of alkalies or other chemicals to dissolve the grease.

The fine gold which is contained in pyrite does not readily unite with mercury. This gold is mostly recovered with the pyrite concentrate or leached by cyanidation. Tellurides do not give up the contained gold to direct amalgamation. The roasting of pyrite and tellurides improves the condition for amalgamating the gold but does not insure a high recovery.

Silver-plated copper plates are generally used to recover the free gold from ore by amalgamation processes. For best results the plates must be kept as clean as possible. Mercury is worked into the surface of the plates until there is exposed a bright pasty amalgam that readily retains the gold as the ore pulp flows over the surface. To effect amalgamation each particle of gold must come into contact with the mercury; the attempt is made to accomplish this by passing the crushed ore and water in a thin layer over the entire surface of the plates and providing for a drop from each of the plates in the series.

Good amalgamation conditions are easily reversed by the careless introduction of oils and grease. Soluble sulphides in the ore will darken the mercury and lower the gold recovery. Arsenic and antimony are particularly harmful. Many of the base metals unite with the mercury to form base bullion, or, even worse, cause the mercury to slough off and carry away precious metals.

Placer gold refers to that gold obtained from alluvial deposits which may have been formed by stream, river, or by the action of water along beaches, and usually includes all gold-bearing loose sand or gravel. The recovery of placer gold is usually made by drift mining on bed rock, hydraulic mining, or dredging.

Since its introduction, the cyanide method of recovering

fine gold from silicious ores has been the usual method employed for the recovery of gold lost in amalgamation processes. In many recently established plants, cyanidation has entirely supplanted amalgamation for the recovery of fine gold.

Before expending any considerable time or money in attempting gold recovery, each prospect should be examined and approved by a competent operator familiar with the particular class of mining necessary, or by a competent mining engineer and metallurgist.

LARGE QUANTITIES OF SURPLUS MILITARY EXPLOSIVES USED INDUSTRIALLY.

More than 126,000,000 pounds of TNT and other surplus military explosives accumulated by the Government at the close of the World War have been used for industrial purposes, states Dr. Charles E. Munroe, Chief of the Explosives Division, United States Bureau of Mines, Department of Commerce. These explosives have been expended in road building, in construction of dams and reservoirs, in draining swamps, in clearing cut-over lands, and for other useful purposes which have added materially to the wealth of the Nation.

After the entrance of the United States into the World War, the Nation set about the production of military explosives on a scale never before undertaken, and this work continued with such acceleration that when the Armistice was declared the country was producing military explosives in quantities never before realized. There is little doubt, Dr. Munroe points out, that this developed capacity was a material factor in ending the War.

A consequence, however, was the accumulation at various points in this country of enormous stocks of explosives, whose safeguarding during storage and transportation constituted a serious and costing obligation, while entailing a menace to the communities near which the material was stored.

The Bureau of Mines, which had taken an active part in the technical research necessary to the production of military explosives on a tremendous scale, advocated the use of these great stocks of surplus explosives on governmental and industrial peace-time projects, pointing out the heavy expense that would be entailed for either the continued preservation

or the destruction of these explosives. This suggestion met with much adverse criticism, great stress being laid on the fact that military explosives, and particularly, TNT never had been used industrially and were, therefore, unfit for such purposes. The Bureau, however proceeded to demonstrate the suitability of these explosives for industrial purposes and issued several publications setting forth the results it had obtained in practice in the field and giving detailed instructions as to the best methods of use. The result was that this huge store of military explosives, instead of being a total loss or constituting a menace to public safety, was diverted into useful peace-time purposes.

Recently the Bureau was informed that some 250 tons of this military TNT, stored for use near an important Federal project, had deteriorated into a dangerous condition and should be destroyed. An explosive expert was detailed to inspect this supply. When tested, both at the Bureau of Mines Experiment Station at Pittsburgh, and at Picatinny Arsenal, this suspected TNT was found to conform completely with the specifications under which it was originally purchased and to be in a perfectly stable condition. This TNT had been packed loose in wooden boxes and with time some of these boxes had become warped and broken, making it necessary to repack such material if it is to be offered as freight.

The expert who examined this TNT reported that it was in first class condition and entirely suitable for use as a blasting agent. It is gratifying, concludes Dr. Munroe, to find that TNT, which has disclosed such admirable qualities for use in blasting, has now been shown, from this test of storage for upwards of ten years, to have excellent keeping qualities also.

CARBON MONOXIDE FROM AN AUTOMOBILE ENGINE USING ETHYL GASOLINE.

PUBLIC interest in atmospheric pollution by automobile exhaust gas, particularly with respect to the ventilation of vehicular tunnels, makes it desirable to ascertain whether the use of modern automobile fuels is tending to change the amount and composition of the products of combustion. The

Bureau of Mines, in coöperation with the Ethyl Gasoline Corporation, has completed tests to determine whether any significant difference exists in the carbon monoxide content of the exhaust gas emitted by an automobile engine when its fuel is changed from ordinary gasoline to the same gasoline containing ethyl fluid (the active ingredient of which is tetraethyl lead) all other operating conditions remaining the same. The results of these tests, which included carburetor adjustments ranging from "rich" to "lean" with engine operating at three-quarter to full load, show no significant change in the composition of the exhaust gas upon changing the fuel supply from ordinary gasoline to the same gasoline containing tetraethyl lead. This was true for tests in which detonation or "knock" was evident as well as for tests in which no detonation was audible. Also, no significant difference in the amount of carbon monoxide per horse power was noted. From a practical health and safety standpoint the amounts of carbon monoxide produced at a given condition of operation were the same regardless of the fuel used.

INERTS IN COAL.

AN investigation has just been started at the United States Bureau of Mines Pittsburgh Experiment Station on the effect of inerts in coal on its utilization. This work, which is being done in coöperation with the Carnegie Institute of Technology and the Pittsburgh Coal Company, will deal primarily with the effect of "fusain" or mineral charcoal on coking properties of coal and with the effect of ash constituents on clinker formation. Several mines of the Pittsburgh bed have been sampled and the coal benches are being analyzed separately for distribution of inert matter. Coking properties of the various benches will be determined and the softening points of the ashes as well.

USE OF MEGGER CIRCUIT IN GEOPHYSICAL PROSPECTING.

THE increasing interest in the use of electrical or other geophysical methods for the location of underground ore deposits has led the United States Bureau of Mines, to study the possibilities of the use of the megger circuit for the measure-

ment of the variation of ground resistivity, an important factor in this means of mineral prospecting.

The megger may be used to measure the various average resistivities at different depths from the surface or to compare the various average resistivities at the same depths. With its use it is possible to locate the direction of the dip and strike of a concealed intrusion, such as a body of ore. About three sites or areas can be measured in one day to a depth of 600 ft., by a crew of three men.

The megger as it is built at present is not entirely suited for geophysical prospecting, states Dr. F. W. Lee, in Technical Paper 440, just issued by the Bureau of Mines. Various improvements which would help in the operation of this instrument in the locating of underground ore bodies are summarized in Technical Paper 440.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting, Board of Managers, September 12, 1928.)

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MR. C. LAURENCE WARWICK, Secretary-Treasurer, American Society for Testing Materials, 1315 Spruce Street, Philadelphia, Pa.

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MR. BENJAMIN FRANKLIN AFFLECK, President, Universal Portland Cement Company, 208 South LaSalle Street, Chicago, Illinois.

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MR. L. J. BUTTOLPH, Electrical Engineer, 410 Eighth Street, Hoboken, N. J.

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MR. R. J. WENSLEY, Electrical Engineer, Westinghouse Electric and Manufacturing Co., East Pittsburgh, Pa.

STUDENT.

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MR. JOHN E. ZIMMERMANN, Day and Zimmermann, Inc., 112 North Broad Street, Philadelphia, Pa.

LIBRARY NOTES.

RECENT ADDITIONS.

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BOOK REVIEWS.

TECHNICAL DRAWING, a manual for evening classes and junior technical schools. By George Edwin Draycott, Wh.Ex., A.M.I.Mech.E. Head of the Mechanical Engineering Department, Borough Polytechnic Institute, London. viii–232 pages (18 x 11 cm.), cloth. New York, Oxford University Press, American Branch, 1927. Price, $2.

There is no branch of manufacture or construction which does not require for its efficient prosecution men, and often a large corps of men, with varying degrees of technical qualification, who are acquainted with approved methods of depicting the objects to be produced or erected, their details, supplying their dimensions and all other information which is required by the workman in the shop or in the

field. There is much for the student to learn in the principles of draftsmanship, the methods of descriptive geometry, upon which technical drawing is based, a degree of manual dexterity and the technique of arranging the plans depicted in accordance with adopted practice as well as an irreducible minimum of previous training in elementary mathematics and a more or less diversified practical knowledge. These requirements relate to the achievement of draftsmanship, only, apart from purely technical qualification which is often, and in fact generally, expected of the modern draftsman.

Technical schools of university grade include more or less elaborate courses in drawing in their curricula, but many of the draftsman who occupy positions in the ranks of the calling are recruited from evening and vocational schools which have come into existance in modern times. "Technical Drawing" is addressed to the latter class. Its purpose, the author states, is to provide a method of instruction which has been found satisfactory in actual practice over a long term of years. The usual topics are covered; instruments and materials, scales, geometrical construction, projections, intersections and developments, sketching, isometric drawings and the technique of lettering, dimensioning and shading drawings.

The making of technical drawings is not a deductive art, and no matter how well versed in the principles of descriptive geometry and geometrical construction the student may be, he will not achieve proficiency without contact with the methods of professional practice and illustrations, which are lacking in this book; professional drawings executed in the best possible manner are necessary for the student to acquire an acquaintance with what he will be expected to produce in the engineering drawing office. There is much of value and practical utility in the book, but as representative of approved practice both in descriptive matter and quality of illustrative example, it falls far short of representing the requirements of modern draftsmanship.

LUCIEN E. PICOLET.

GLYCEROL AND THE GLYCOLS. Production, Properties and Analyses. By James W. Lawrie, Ph.D. 447 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1928. Price, $9.50.

The fact that a volume of over 400 pages can be devoted to one compound shows vividly the enormous development of applied chemistry. Glycerol has been known for many years. It was obtained first by Scheele. It is by the way stated in this book that Chevreul called it "glycerol" but this seems improbable. The form of the name, strictly correct according to modern organic nomenclature, is of comparative recent use. It seems certain that Chevreul must have called it "glycérine," the French form. Actually, the name is an error in so far as it was merely given on account of slight sweetness, and it was often termed in time past "the sweet principle of fats." It is not contained in fats except in minute amount and in rare cases, but it is produced from the fats in the process of manufacture. When it first became a familiar article about the middle of the last century, its viscosity led to its being assumed to have some emollient action on the skin presumably a view aided by its being derived from oils and fats. In the scientific development or organic chemistry it was definitely recognized as an alcohol, the

proper termination given to it and it is no longer considered as suitable for application to the skin.

The manufacture of glycerol is now a very important industry, among others is its extensive application in the manufacture of the well known high explosives. For this purpose it must be of high purity.

In preparing this work the author has brought together in one compact well-printed volume all the important data concerning the manufacture, purification, and analytic control. An interesting summary of the early history is given and the several methods by which the substance is obtained are fully described. Much space is given to the production by fermentation. The physical properties and constants are set forth in detail, also the various reactions, the analytic methods and the international standards, and specifications. A chapter is devoted to the manufacture of nitroglycerin termed here in deference to the exact nomenclature "nitroglycerol." About thirty pages are devoted to the glycols among which, of course, one of the most familiar forms, ethylene glycol, is fully described. A brief chapter on the future of "glycerol" ends the main part of the text. In addition to the use for the manufacture of an explosive glycerol has been extensively employed as an anti-freeze for automobiles. It is stated that in 1926 over thirteen million pounds were used for this purpose. Perhaps the most interesting feature of glycerol production is that by fermentation. German chemists during the war perfected this method so that about one thousand tons were produced per month.

HENRY LEFFMANN.

ISOSTASY. By William Bowie, Chief, Division of Geodesy, U. S. Coast and Geodetic Survey, President, Section of Geodesy, International Geodetic and Geophysical Union. XIV–275 pages, 39 illustrations, 12mo. cloth, New York, E. P. Dutton & Company. Price, $5.00.

The word isostasy made its first appearance in a paper by Dutton, "On Some of the Greater Problems of Physical Geology" delivered before the Philosophical Society of Washington on April 27th, 1889. He would have preferred the word, "isobary" but it already had been preëmpted by the meteorologist. At the time of its inception Dutton employed the term "isostasy" to designate the state of equilibrium attained by a heterogenous body such as the earth under the influence of gravity and rotational stresses. Although isostasy retains to this day the meaning attributed to it by Dutton, numerous theories on just how the isostatic state is produced have arisen in later years and which seem to provide a more logical explanation than that set forth by Dutton. In the chapter on the development of the isostatic theory the author lays particular emphasis upon those of Airy and Pratt. Both these theories are elaborated to a greater extent in the chapters following. According to the author the Pratt theory seems to be the more acceptable. This theory postulates the earth's crust to be of uniform depth but composed of materials having a varying density. On the other hand, Airy's theory calls for a crust of uniform density and of varying thickness or depth.

The establishment of the existence of isostasy in the earth's crust has served to cast shadows of doubt over some of the earlier theories explaining the configuration of our terrestial sphere. An explanation for the disagreement between these

earlier and the present theories is found in the fact that the former were founded upon geological observations while the latter have been deduced from geodetic data. These geodetic data which chiefly consist of deflections of a plumb-bob from the vertical and gravity anamolies have been determined with a high degree of accuracy and are responsible for establishing beyond reasonable doubt the existence of isostatic equilibrium throughout the earth's crust. It is generally known that the earlier theories attempted to explain the configuration of the earth's surface by assuming a progressive cooling off of the earth with an accompanying shrinkage of the central mass or core. When such occurred the relatively rigid crust would tend to shrink, such a tendency resulting in a collapse of the crust at its weakest point and a subsequent pushing up or elevation of crustal matter due to pressure upon it in a horizontal direction. A close study of isostatic conditions indicate that such is not the case. All evidence leads to the assumption that the rising of portions of the earth's crust is caused by forces exerted in a vertical direction, any horizontal movement of matter being confined to plastic subcrustal material. As a corollary to the isostatic theory and contrary to the older theory the earth's crust is assumed to be relatively weak due to the fact that isostatic equilibrium of the crust is acquired readily even in comparatively small areas.

Evidently the author has taken pains to make his subject readily visualized and understood by the readers who are meeting for the first time this concept of isostasy. The liberal use of diagrammatic figures is particularly commendable. The reader should not be discouraged if he or she fails to grasp all details of the theory in the first chapter or two. This first portion is introductory and historical in nature and the various passing references are elaborated thoroughly in ensuing chapters. At the end of chapter eight the author has furnished a bibliography of geological literature relevant to his subject.

T. K. CLEVELAND.

SOLUBILITIES OF INORGANIC AND ORGANIC COMPOUNDS. A compilation of quantitative solubility data from the periodical literature. By Atherton Seidell, Ph.D. Supplement to the second edition containing data published during the years 1917–1926 inclusive. 568 pages, 8vo. New York, D. Van Nostrand Company, 1928. Price, $8.

This edition covers the whole alphabet beginning with "Acenaphthene" and ending with "Zirconium sulphate." There is in addition a comprehensive detailed index and list of authors. Comment on the book would merely repeat what has been said of the previous edition. The preparation of such a work is a great labor and draws heavily on the patience of the author, but the data are of great use to workers in chemistry and physics. The style of figure used is satisfactory and the arrangement of the data is in serviceable form. The main purpose of the present volume was to include many data not available when former edition was published. To republish the original volume with corrections and additions required by the progress of research would have involved very great cost, and the plan of issuing a supplementary volume is entirely commendable. The method of presenting the data and the form in which these are set forth are excellent.

H. L.

AMERICAN CHEMISTRY. A record of achievement, the basis for future progress. By Harrison Hale, Ph.D. Second Edition, ix–255 pages, illustrations, 8vo. New York, D. Van Nostrand Company, Inc., 1928. Price, $2.50.

This being the second edition needs but passing notice. The author says properly that the years that have passed since the issue of the first edition have demonstrated the great value of chemistry in the world's affairs, and especially its importance in the United States. The record of American chemistry while creditable to those who have taken part in it was for many years somewhat inferior to that of a few other countries. It is a trite observation that Germany was the leader in this field and this was due to the equally cultivated theory and practice. Americans have shown great progress in fields in which practical work is dominant phase, such as machinery of all types, but in sciences in which theory is important, or in which the work must be done for the sake of the knowledge required without necessarily resulting in practical benefit America was for a long while inactive. Matters have greatly improved of late and excellent records are being made in the laboratories of this country. Concerning the book in hand the reviewer feels obliged to disapprove of the highly surfaced paper on which it is printed. This is due indeed to the numerous illustrations, but many of these could be omitted without loss of value to the book as they represent scenes and apparatus quite familiar to educated readers. There seems also to be a disproportionate attention given to those chemists who have accomplished practical results as compared with workers who have developed the field of pure chemistry.

H. L.

PROBABILITY AND ITS ENGINEERING USES. By Thornton C. Fry, Ph.D. Member of the Technical Staff, Bell Telephone Laboratories, Inc. xiv–476 pages, 8vo, cloth. New York, D. Van Nostrand Company, Inc., 1928. Price, $7.50.

Those who would achieve an applicable knowledge of the theory of probability will find themselves confronted with a task which demands the exercise of the most subtile logic as well as the acquisition of a wide diversity of analytical accomplishment. The effort is worth the while, the rapid strides of physical science have been made in part by enlisting its aid and the many new applications of science which have entered into the scheme of modern life have given rise to a host of new problems in which, again, solutions can only be effected by the methods of the theory of probability. Since the time of Fermat and Pascal, nearly every mathematician of note has made some contribution to the subject, perhaps because it has proved slow to yield a satisfactory general basis for the deduction of applicable formulation. The modern development of the theory has been in general taken upon a statistical basis rather than upon a foundation of pure and priori logic, but the author expresses a conviction that probability is not an experimental science.

Laying great stress upon logical considerations and utilizing very fully the multiplicity of mathematical devices which are the bequest of two centuries of mathematical genius, Dr. Fry has produced a treatise of a high quality of didactic value which deals deductively with fundamentals, derived theorems, and their application to typical problems of certain modern industries. The author is unsparing of explanation in analyzing the conditions and appraising the logic of a variety of problems which he introduces in unfolding the intricacies of the subject

into practical working methods, as well as generous in supplying in detail the steps in the derivation of formulas. Practical working methods, it may be noted, are of special interest in this subject which until recent times has often been more one of mathematical abstraction than one of practical utility except in the actuarial and financial fields and in the adjustment of observations by the widely used method of least squares.

The beginning of the subject is discussed in much the same sequence as is ordinarily found in texts on probability but it differs markedly from them in dwelling at length upon the logic of every theorem considered, and in the diversity of types of problems examined. In this part is included an illuminating explanation of the eulerian integral for determining the value of factorials which is later used in the derivation of the Stirling formula for the approximate value of the factorials of large numbers. Experimental probability with relation to Bernoulli's Theorem and Baye's Theorem and, further on, Distribution Functions occupy many pages of analytical and critical discussion. As is to be expected of an author who is a member of the technical staff of the Bell Laboratories, whose contributions to scientific literature are of a uniformly high order of excellence, many of the problems which are considered at length are those which arise in telephone service or in the functioning of telephonic equipment. Most of these problems, the development of which entails a variety of formidable analytical transformations, necessarily premise a familiarity with such equipment. The volume concludes with the application of probability to the kinetic theory of gases which is presented with its thoroughness characteristic of other parts of the volume. An appendix containing a number of tables including tables of factorials and logarithms of factorials, binomial coefficients and error functions. The work is notable in setting forth at great length the mathematical and logical complexities of modern applications of the theory of probability in unusually readable form quite in contrast with the terse and abstract language usual with the mathematician.

L. E. P.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 289. Forces on Elliptic Cylinder in Uniform Air Stream, by A. F. Zahm, R. H. Smith, and F. A. Loudon. 18 pages, illustrations, quarto Washington, Government Printing Office, 1928, price ten cents.

This report presents the results of wind tunnel tests on four elliptic cylinders with various fineness ratios, conducted in the Navy Aerodynamic Laboratory, Washington. The object of the tests was to investigate the characteristics of sections suitable for streamline wire which normally has an elliptic section with a fineness ratio of 4.0; also to learn whether a reduction in fineness ratio would result in improvement; also to determine the pressure distribution on the model of fineness ratio 4.

Four elliptic cylinders with fineness ratios of 2.5, 3.0, 3.5, and 4.0 were made and then tested in the 8 by 8 ft. tunnel; first for cross-wind force, drag, and yawing moment at 30 miles an hour and various angles of yaw; next for drag at 0° pitch and 0° yaw and various wind speeds; then for end effect on the smallest and largest models; and lastly for pressure distribution over the surface of the largest model at 0° pitch and 0° yaw and various wind speeds. In all tests, the

length of the model was transverse to the current. The results are given for standard air density, $\rho = .002378$ slug per cubic foot.

This account is a slightly revised form of Report No. 315, prepared for the Bureau of Aeronautics, July 13, 1926, and by it submitted for publication to the National Advisory Committee for Aeronautics. A summary of conclusions is given at the end of the test.

COLLOID CHEMISTRY. By T. Svedberg. 2d edition, revised by Arne Tiselius, 8vo., 302 pages, numerous illustrations. The Chemical Catalog Co., New York, 1928. $5.50.

This is one of the volumes of the series of monographs issued by the American Chemical Society and now constituting a large and valuable library of chemical literature. Being a second edition there is little to be said except to announce publication. The text is a report of lectures delivered at the University of Wisconsin in 1923. It should be added, however, that the revisor has added much new matter and carefully scanned the original material. The volume therefore includes a very large amount of information on colloid chemistry, the growth of which has been so striking a feature of the last quarter century in chemical research. Some historical data are given in the introductory portion, in which there seems to be a somewhat invidious allusion to Graham's work in so far as to enumerate some anticipations of him. Adumbrations of this character are common and may be made concerning almost all epoch-making discoveries or inventions. Da Vinci is sometimes given credit for initiating aviation, Lucretius has been thought to allude to motion pictures, and recently assertions have been made that wonderful anticipations of modern knowledge were stated by Roger Bacon. Graham introduced the word "colloid" into chemical literature and in his paper in *Philosophical Transactions*, stated that he regarded the difference between colloids and crystalloids as due to molecular differences. English readers will be impressed with the fact that the text was written in fairly good English by the Swedish author, but at the same time it would have been proper for an English speaking editor to have looked over the MS.

HENRY LEFFMANN.

RADIO. A study of first principles for schools, evening classes and home study. By Elmer E. Burns, Instructor in Physics, Austin High School, Chicago. xv–255 pages, illustrations, 20 x 13 cm., cloth. New York, D. Van Nostrand Company, Inc., 1928. Price, $2.

Since radio has become an established art of great importance, many books dealing with the subject have become available, most of them adapted to the needs of particular types of readers, sometimes too mathematical for any one but a college-trained person, again too rudimentary to convey applicable information. Professor Burns has found a very happy middle course. He begins at the beginning by first describing simple crystal sets, one-tube sets and amplifiers with enough quantitative data for a resourceful reader to build a set. In this part of the programme doubtless many of his readers will have already qualified. It is, indeed, interest of that sort which is often the beginning of a technical education and a life work. This is preliminary to what may be called, for want of a more suitable term, a proximate analyses of the subject which conveys a large amount of

satisfactory information on the principles and operation of the component elements of receiving sets.

The aim of the work is to present simply and clearly the fundamental principles of electricity and magnetism which it does without at any point losing sight of their application to radio. First we have electric batteries, the magnetic action of an electric current, and electric circuits and Ohm's law, which cover three chapters. From that point we deal wholly with radio in chapters on electron tubes, alternating currents, detectors and amplifiers, fundamentals of receiving circuits, oscillating and transmitting circuits, radio measurements and an appendix containing useful formulas and symbols.

It is difficult to say on what particular topic the book is at its best for their treatment is one of harmonious adequacy in their inter-relationship. The chapters on thermionic tubes will claim a goodly share of approval for their clearly expressed and well illustrated account of their properties, but other parts of the book compare very favorably with these on all matters of text-book performances. A conscientious study of this book insures the acquisition of sound and practical knowledge in applied radio and an excellent preparation for one who contemplates an exhaustive course of study in radio engineering.

LUCIEN E. PICOLET.

BALISTIQUE INTERIEURE THÉORIQUE ET TABLES NUMERIQUES. Par G. Sugot, Professeur a l'Ecole d'Application de l'Artillerie Navale. xxx–130 pages, illustrations, 25 x 14 cm., paper. Paris, Gauthier-Villars et Cie, 1928. Price, 40 francs.

BALISTIQUE EXTÉRIEURE THÉORIQUE. Par G. Sugot, Professeur a l'Ecole d'Application de l'Artillerie Navale. iii–94 pages, illustrations, 25 x 14 cm., paper. Paris, Gauthier-Villars et Cie, 1928. Price, 25 francs.

The design of a piece of artillery, like any other apparatus with constrained moving parts upon which certain determinate forces act, is purely an engineering problem in which the specifications of bore, rifling, weight of projectile and muzzle velocity are the fundamental data chosen within the possibilities of sound practice. Once the contour of the powder chamber and bore are determined, the process of mechanical design may proceed. Interior ballistics deals with all the factors which enter into the determination of these relatively simple dimensions. These factors are many and involved and entail thermodynamic considerations and experimental investigation of the properties of the powders used as a propellant, the work of whose varying pressure upon the base of the projectile, generally stated, must be equivalent to its kinetic energy at the muzzle plus resistances and losses during its transit through the bore.

These processes of coördinating the results of experiment with theoretical considerations constitutes the subject-matter of this specialized branch of ordinance engineering. The present volume which is an exposition of these theoretical and experimental methods are included in the course of study at the Ecole d'Application de l'Artillerie Navale. The work is in two major divisions, the motion of the projectile in the bore, and numerical processes. The first comprises equations of interior ballistics, ballistic analogies, constant emission powders, variable emission powders, summary and conclusions. The second part comprises

a general survey, determination of the characteristics of B-powders, theoretical derivation of working formulas of Ch.S., derivation of the experimental working formulas of Dupuis, practical value of the working formulas, certain constants derived from theoretical working formulas. The numerous tables which are vital to a subject of this kind are placed at the beginning of the volume. Particularly for the use of foreign readers a key to symbolization which does not appear to be included would be decidedly helpful, especially if the book is used for reference. The work is a highly analytical exposition of modern methods in interior ballistics.

As distinguished from a consideration of the forces which act upon the projectile before it reaches the muzzle of interior ballistics, the problems of analysis of the behavior of the projectile after leaving the muzzle, of exterior ballistics, are varied and difficult and their solution likewise enlists the coördination of experiment with theory. General Charbonnier, the author of a comprehensive treatise on the subject has aptly termed that branch of the art "terrestrial astronomy."

The present volume is devoted to the theoretical and deductive processes of the subject. According to the paging of the volume from p. 827 to p. 920 and a foot note referring to volumes I and II of Traite de Balistique Exterieure by the same author, it presumably forms a part of that work though there is no indication on the title page of any relation to those volumes. The five divisions of the work cover: ballistics of a material point; motion of the projectile about its center of gravity; influences affecting trajectory; practical methods of ballistic computation. Like in its companion work, the analytical deductions are very fully developed.

L. E. P.

MARVELS OF MODERN MECHANICS. The Mastery of Land, Sea and Air. By Harold T. Wilkins. xi–280 pages, 21 x 14 cm., illustrated, cloth. New York, E. P. Dutton & Company. Price, $3.

Almost daily some new achievement in applied science is being brought to our attention. The list is already a long one, and it continues to grow until the mind becomes confused in endeavoring to recall the essential details of even the more important ones. Several decades ago we had the deservedly popular books of adventure of Jules Verne whose stories were woven about a framework of applied science with rarely if ever a fault of technique, with the possible exception of a novelist's license to strain a scientific fact to effect a suitable denouement. Since that day, applied science has become interesting enough *per se* to require no background of fiction to enlist interest, and several books descriptive of a variety of "wonders of science" and the like have since that time made their appearance. Mr. Wilkins steps into the field to bring these matters thoroughly up to date and the term "mechanics" in this title is in a rather comprehensive sense in that the foundation of physical science rests upon the principles of mechanics. However the book deals almost entirely with purely mechanical developments, though an exception here and there occurs as for instance in the mechanics of the telegraphic transmission of pictures and in dealing with phenomena of electricity and light and other radiations, and some chemical matters.

As the author states, the book is a summary of modern achievement in the laboratory and experimental science and wherever power is used. A complete summary of such a scope would require several volumes like the present, but the

collection is very extensive and is representative of many devices and methods of a scientific character which at the time of their inception created widespread interest and sometimes amazement at their revolutionary character. Among these it is only necessary to mention the X-ray, aerial navigation, applications of the gyroscope, wireless and radium, the extensive applications of which have now become familiar. Many other topics of deep and timely interest are discussed, such as salvaging sunken ships, submarine cable laying, archeological excavation and scenic effects in the theatre. The discourse is informal, precise and highly informative. There is no index and names of inventors and dates are not as generously supplied as might be desired in a book of reference. However as a means of entertainment for those who desire a deeply interesting account of representative achievements in applied science, the book very adequately meets that requirement.

LUCIEN E. PICOLET.

PUBLICATIONS RECEIVED.

American Chemistry, by Harrison Hale, Ph.D. Second Edition. 255 pages, illustrations, 8vo. New York, D. Van Nostrand Company, Inc., 1928, price $2.50.

Glycerol and the Glycols Production, Properties and Analyses, by James W. Lawrie, Ph.D. American Chemical Society Monograph Series. 447 pages, illustrations, tables, 8vo. New York, The Chemical Catalog Company, Inc., 1928, price $9.50.

A First Course in Physics for Colleges, by Robert Andrews Millikan, Ph.D., Henry Gordon Gale, Ph.D., and Charles William Edwards, M.S. 676 + XLII pages, illustrations, plates, 8vo. Boston, Ginn and Company, 1928, price $3.72.

An Epitome of Swedenborg's Science, by Frank W. Very, S.B. 2 volumes, plates, portrait, 8vo. Boston, The Four Seas Company.

Colloid Chemistry by The Svedberg. Second Edition revised and enlarged by Arne Tiselius. American Chemical Society Monograph Series. 302 pages, illustrations, diagrams, 8vo. New York, The Chemical Catalog Company, Inc., 1928, price $5.50.

National Advisory Committee for Aeronautics. Technical Notes, No. 294, Wind tunnel force tests in wing systems through large angles of attack, by Carl J. Wenzinger and Thomas A. Harris. 7 pages, diagrams, quarto. No. 295, The effect of tip shields on a horizontal tail surface, by Paul V. Dronin, Earl I. Ramsden and George J. Higgins. 10 pages, tables, diagrams, quarto. Washington, Committee, 1928.

Introduction to Modern Physics, by F. K. Richtmyer. 596 pages, illustrations, 8vo. New York, McGraw-Hill Book Company, 1928, price $5.00.

A Text-Book of Inorganic Chemistry, edited by J. Newton Friend, D.Sc. Volume VI, part I, Nitrogen, by Edmund B. R. Prideaux and Herbert Lambourne, 242 pages, illustrations, 8vo. Volume X, The Metal-Ammines by Miss M. J. Sutherland. 260 pages, 8vo. London, Charles Griffin and Company, Ltd., Philadelphia, J. B. Lippincott Company, 1928.

CURRENT TOPICS.

High Angular Velocities Obtained by Rotors having no Solid Axis. E. HENRIOT AND E. HUGUENARD. (*Jour. de Phys. et Rad.*, Nov., 1927.) Hitherto we have had rotors with rigid shafts and, on the other hand, rotors with flexible axes or flexible bearings. Flexible shafts have the advantage of taking care of errors of centering by bending so that the center of gravity of the rotor falls upon the axis of rotation when the critical speed is exceeded. Nevertheless it has been the rigid shaft to which recourse has been generally had when high speeds of rotation have been required as, for example, for Foucault mirrors. Perfection of centering has been aimed at but the results have been none too good in spite of the high costs and difficulties of construction. Now the authors have devised rotors that are supported by sheets of gas and that can be rotated an incredible number of times per sec. They began by working with small rotors. Their success in this field led them to try to use rotating parts weighing several kilograms. Here also their method worked well.

A jet of gas escaping upwards from a conical orifice supports a steel ball at a distance of several cm. from the opening. The stability of the ball is, however, slight. The authors found that the ball is very stably supported when it is only a fraction of a mm. from the sides of the aperture. If it be placed a few mm. from the sides, it is drawn in close even if the gas be flowing out under the pressure of 100 atmospheres. In the experiments of this paper the gas not only supports the rotor but also makes it rotate. The stator is a hollow cone into which discharge air or carbon dioxide arriving through pipes under pressures ranging from 1 to 6 kg. per sq. cm. The tubes make an angle of 45° with planes tangent to the cone. The rotor also is conical in form, though the angle of its cone is greater than that of the stator. With a rotor 45 mm. in diameter 1,300 revolutions per sec. were obtained with a peripheral speed of 185 meters per sec. With a rotor 11.7 mm. in diameter the corresponding results were 11,000 r.p.s. and 404 m./sec. It is difficult to comprehend that the spinning part, less than half an inch in diameter, revolved so fast that a point on its circumference was travelling at the rate of 840 miles an hour. Later small steel mirrors were mounted on the small rotor but they reduced the speed greatly. Even with mirrors only 4 or 5 mm. on a side the churning of the air kept the number of revolutions down to about

5,500 per sec., but even so measurements are being made on the velocity of light over a distance as small as one meter. It was discovered that stability and certainty of action were increased by using heavier rotors, and application was made to the driving of centrifuges where the containing cylinder was either on top of the rotor or hung beneath it, the rotor itself being under the stator and supported there by the suction of the escaping gas.

One advantage of the gas bearing is that parts can be used for rotation that are only roughly centered. In one instance a weight of 100 g. was put 2 cm. from the axis and the whole made to rotate 1,000 times per sec. The mass of the unaugmented rotor seems to have been about 700 g. In such cases it is advisable to support the stator elastically so that the rotor out of center may not strike the fixed part.

The rate of revolution was measured streboscopically, by acoustical methods and by mechanical means. The authors point out that with a rotor of 6 mm. radius making 10,000 r.p.s. the centrifugal force at the periphery amounts to about 2,400,000 times the force of gravity, and that in the 6 mm. from circumference to center this force dwindles to zero. In centrifuges carrying discs of lead the centrifugal force was 150,000 that of gravity and the cold lead in a few minutes flowed away from the middle and heaped up at the circumference. G. F. S.

The Electrodeless Discharge through Gases. SIR J. J. THOMSON. (*Proc. Phys. Soc.*, London, April 15, 1928.) "By 'electrodeless discharge' I mean a discharge where the currents form closed circuits in the gas, and have not to pass from gas to metal, as in discharge tubes with metallic electrodes, or from gas to glass, as when metallic electrodes are placed outside the tube. This ring discharge can be produced by placing outside the bulb or tube containing the gas at a low pressure a solenoid placed in a circuit connecting the outsides of two Leyden jars. The insides of these jars are connected respectively with the two terminals of an induction coil; when sparks pass between these terminals, rapidly alternating currents, whose frequency is inversely proportional to the square root of the product of the capacity of the jars and the self-induction of the circuit, pass through the solenoid. The electromagnetic induction due to these currents starts in the exhausted vessel currents which flow in rings whose axis coincides with that of the solenoid." For a long time it has been recognized that letting light fall on a spark-gap makes it easier for the spark to pass. This is dueto the effect of the light on the negative terminal whereby electrons are set free. The electrodeless discharge affords excellent

opportunities for examining the effect of light and other agents upon discharge through gases. The discharge tube was spherical in form and had opening into it two long tubes one the prolongation of the other. Light entered one tube through a quartz window, passed through the spherical space and made its exit by the second tube, so that of all the apparatus only the ends of the two tubes were struck by the light. The voltage of the induction coil was reduced until the ring discharge just ceased. The passage of the light was followed by the appearance of the discharge. This effect takes place in air, hydrogen, helium, argon, oxygen, nitrogen, carbonic acid, iodine vapor and mercury vapor. It is not as pronounced at low pressures as at somewhat higher ones. It has been observed in air for pressures ranging from .007 to 7 mm. The effect is caused by light from various sources, from a mercury lamp, an arc lamp, from a spark between metal terminals or from a brush discharge. A spirit lamp gives light that produces the result as does a Bunsen burner or an oxyhydrogen flame. To be effective the light must not pass through anything that absorbs ultra-violet light. Thin films of glass or mica do this and keep the ring discharge from occurring, while quartz and rock salt let the discharge appear. "The effect of illumination on the ring discharge is evidently due to the absorption of ultra-violet light. The energy absorbed, though not sufficient to eject electrons from the atoms or molecules of the gas, may be sufficient to put these for a time, which is not infinitesimal, into a state in which they are more easily ionized than when they are normal. Thus, as the work of ionisation is already half done, the external electric field will not be called on to supply as much energy to the free electrons to make them ionize these excited molecules as would be required to make them ionize normal atoms or molecules; and thus the ring discharge will pass more easily when the gas is illuminated than when it is not."

Interesting effects due to impurities in the gas have been observed. Furthermore the electrodeless discharge makes the gas active chemically and causes it to form unexpected compounds. For example, the discharge was formed in oxygen in a tube that contained MgO. The light from the discharge makes the magnesia phosphoresce. It was noted that the pressure of the oxygen fell quickly. Oxygen having a volume equal to many times that of the MgO was introduced and disappeared. The compound had the same appearance but that it had really changed was shown by its ceasing to phosphoresce. He concludes that "oxygen through which the ring discharge is passing combines with MgO to form a higher oxide. I understand that a higher oxide of magnesium is manufactured in considerable quantities, though it is not mentioned

in the majority of text books on chemistry." By the same method higher oxides of calcium and zinc were produced. G. F. S.

The Origin of the Aurora Borealis. E. O. HURLBURT. (*Terrestrial Mag.*, March, 1928.) "A theory of the aurora borealis has been developed by Birkeland, Stoermer, Vegard, and others on the assumption that the aurora is caused by charged particles emitted from the Sun which under the influence of the magnetic field of the Earth are diverted to the polar regions. There their energy is given to the atmosphere and by some process is converted partially or totally into the auroral light." There are objections to this explanation from the side of theory as well as from that of experiment. For instance no one has succeeded in reproducing the spectrum of the aurora in the laboratory.

The author ventures to suggest a new theory. The energy that finally appears with auroral light comes from the sun in ultra-violet radiation. In the higher part of the earth's atmosphere it is absorbed with the production of ions and electrons. These particles diffuse but combine to only a slight extent. The ions drift along the magnetic field of the earth and concentrate in the polar regions. In lower levels there they recombine and the energy of combination appears in the auroral light.

From observations made in Maine, August, 1927, the power of the auroral light then seen is calculated to have been 10^{15} ergs/sec. This agrees sufficiently well with the results of another computation based on the rate of flow of ultra-violet light to the earth. The author wisely remarks "But here I must stop, partly to avoid being too rash, but mainly to admit that my contemplation of the aurora has been too limited to give that steadying of ideas which comes only from experimentation." G. F. S.

The Application of Electrical Resistance Measurements to the Study of the Atmospheric Corrosion of Metals. J. C. HUDSON. (*Proc. Phys. Soc.*, London, April 15, 1928.) The effect of atmospheric corrosion upon metals can be traced by a study of the weight of the specimen as time goes on. Vernon has done this "and the accuracy obtainable is well demonstrated by the fact that he was able to follow the development of tarnish films on lead and aluminium in cases where the final weight increment, after 25 days' exposure, was as low as 0.28 milligrammes." Much labor is, however, necessary especially when the specimen is exposed in the open where the products of chemical action may be carried away by rain or otherwise lost. When, therefore, the Atmospheric Corrosion Committee of the British Non-Ferrous Metals Research

Association appointed the author to make systematic field tests he proceeded to devise a method that should be both more accurate and more convenient. He thus developed a previously existing procedure in which the extent of corrosion of a metallic conductor is indicated by the increase in its electrical resistance. As corrosion goes on metal is replaced by non-conducting products of chemical action. Thus the cross-section of the metal grows less and the resistance becomes greater. "Since the resulting percentage increase in the resistance of the specimen is equal to the percentage diminution in the cross-section, the method has the advantage of giving results that are immediately comparable for different materials." In addition specimens of convenient size can be used and the same specimen can be continuously observed.

Thin wires show a greater percentage change in their resistance than do thick ones but the results suffer in accuracy from irregularities of the surface. Hence 18 S.W.G. was the standard size selected. About 200 cm. of the wire was wound into coils. Three such coils of the same metal were exposed to the atmosphere while a fourth control coil was kept in the laboratory. The preliminary cleansing of the metal and the attachment of terminals were well cared for. The resistance was determined by a Wheatstone bridge method. An increase of any per cent. in the resistance means that the effective cross-section has diminished by the same per cent., the assumptions being that the corroded film is uniform in thickness and that there is a sharp boundary between the untouched metal and the corroded parts.

Wires of the same gauge of nickel, copper, copper-nickel, phosphor-bronze, nickel-silver, cadmium-copper, and brass were exposed to the weather for 53 days. Brass showed the greatest change of resistance, from 2.65 per cent. to 3.79 per cent. according to composition. Copper-nickel was least affected, 1.49 per cent. Copper wire was measured at the end of different periods. One specimen after 101 days gained .88 per cent. in resistance, after 213 days, 3.99 per cent. and after 381 days 4.82 per cent. The change of resistance was found to be inversely proportional to the diameter of the wire when short periods of exposure are omitted from consideration. The rate of corrosion was shown to be much greater in winter than in summer.

It is evident that a method of considerable practical utility has been developed and successfully applied. G. F. S.

Some Observations of Atmospheric-Electric Potential-Gradient on Mountain Peaks in the Peruvian Andes. W. C. PARKINSON. (*Terrestrial Mag.*, March, 1928.) The Department of Terrestrial

Magnetism maintains an observatory at Huancayo, Peru. Here on a fairly level plain at an altitude of 11,000 feet continuous registration is made of electric air-potential. In May, 1927, a continuous record was taken on a hill elevated 500 feet above the plain and somewhat later the apparatus was transferred to a ridge 2,200 feet higher than the observatory. A consideration of average values of the gradient shows that both on hill and ridge it reaches a minimum value at 6 A.M. and a maximum at 10 A.M. For the hill the minimum and maximum averages are about 41 and 214 volts per meter while the corresponding values on the ridge were 48 and 337 volts/m. The gradients on the hill were about 2.17 times as great as those at the observatory while those on the ridge were 3.06 times as great. It might have been expected that the difference between the values on hill and ridge would be larger but the contours of the two elevations must be taken into account.

G. F. S.

A Proposition of Leibniz for Extended Observations of the Earth's Magnetism. (*Terrestrial Mag.*, March, 1928.) After the foundation of the Royal Academy of Sciences in Berlin Leibniz addressed a memorial to the king of Prussia, Frederick I, in which he recommended that missionaries should receive special scientific training and should thereafter be sent to Russia, Turkey, Persia, India and China. There in addition to their religious activities they should act both as teachers of science and as investigators. The king was advised to attach to the minister to Russia a young man who had received the preliminary scientific training in the hope that Peter the Great might send him into the provinces to make observations on terrestrial magnetism. The czar's interest in navigation was well known and moreover the Englishman Halley had just made a magnetic chart on which lines ran through places having the same declination. These lines were drawn only on the water areas of the earth and stopped where the land began. Leibniz desired the king to have magnetic observations taken in his own territories and to have them made in the Orient as well through the goodwill and help of the czar, adding "Such an undertaking would be no less glorious for your Royal Majesty and useful for the public than it would be advantageous for the Christian work of missions." At least on the secular side Dr. L. A. Bauer and the Carnegie Institution of Washington have carried the suggestion to a successful issue. G. F. S.

Atmospheric Dust and Condensation Nuclei. R. K. BOYLAN. (*Proc. Royal Irish Acad.*, Vol. 37, A, No. 6.) There is confusion

arising from identifying the dust particles in the air with the nuclei on which water vapor condenses. An Owens jet dust counter was used to get the number of particles of dust in unit volume and an Aitken's Pocket Dust-Counter did the same service for the nuclei. Observations extending over eight months failed to disclose a single instance when the number of dust particles in a cu. cm. was the same as the number of nuclei in the same volume. Indeed the nuclei always vastly outnumbered the dust particles, not infrequently in the ratio of 30 : 1. Rain reduced the number of dust particles and slightly increased the nuclei. Fog greatly augmented the number of both. A correlation coefficient of .73 was calculated for the occurrence of the two types of particles. This points to a common origin.

Observations were made at 15 min. intervals in a room in which a lamp was lighted at 3:30 p.m. and put out at 4 o'clock.

Time	3	3:15	3:30	3:45	4	4:15
Nuclei per cc.	15,600	15,800	16,000	33,000	100,000	34,000
Dust per cc.	—	600	635	1,450	1,540	1,150

The dust content of rooms was artificially varied by sweeping accumulations of dust or by setting free the contents of a vacuum cleaner bag. While the dust count was always very greatly increased, the number of nuclei never increased in the same ratio. Indeed the introduction of vacuum cleaner dirt usually reduced the number of nuclei. Another series of experiments showed that, even when ordinary nuclei have been filtered out of air, dust particles will not serve as centers of condensation. Moreover dust particles and nuclei readily unite so that the setting up of heavy clouds of dust does away with every nucleus previously present.

G. F. S.

Health of Workers in Dusty Trades.—The United States Public Health Service has completed a study of the health of workers in a Portland cement plant, the first of a series covering the dusty trades, according to an announcement recently made by Surgeon General H. S. Cumming. The study was undertaken to ascertain whether persons working in an atmosphere containing numerous minute particles of a calcium dust suffered any harmful effects. The investigation was conducted in one of the older, dustier plants, so that the effect of large quantities of the dust could be observed. Records of all absences from work were kept for three years, and the nature of disabling sickness was ascertained. Physical examinations were made, X-ray films taken, and the character and amounts of dust in the atmosphere of the plant were determined.

The results of this investigation indicated that the calcium dusts generated in the process of manufacturing Portland cement do not predispose workers to tuberculosis nor to pneumonia. The workers exposed to dust experienced, however, an abnormal number of attacks of diseases of the upper respiratory tract, especially colds, acute bronchitis, diseases of the pharynx and tonsils, and also influenza, or grippe. Attacks of these diseases serious enough to cause absence for two consecutive working days or longer occurred among the men in the dustier departments at a rate which was about 60 per cent. above that of the men in the comparatively non-dusty departments. Limestone dust appeared to be slightly more deleterious in this respect than cement dust.

Outdoor work in all kinds of weather such as was experienced by the quarry workers appeared to predispose to diseases of the upper respiratory tract even more than did exposure to the calcium dusts. In the outdoor departments of the plant, also, the highest attack rates of rheumatism were found. The study also indicated that work in a cement dusty atmosphere may predispose to certain skin diseases such as boils, to conjunctivitis, and to deafness when cement dust in combination with ear wax forms plugs in the external ear. When the dust in the atmosphere is less than about ten million particles per cubic foot of air it is doubtful that the above-mentioned diseases and conditions would be found at greater than average frequency.

Modernization of plants and installation of ventilating systems are helping to solve the dust problem of the industry.

Measurement of the Biologically Active Ultra-Violet Rays of Sunlight. L. HILL. (*Proc. Royal Soc.*, A 774.) A standard solution of methylene-blue was made and acetone was added. Ultra-violet light decomposes the acetone and the methylene-blue acting as a hydrogen acceptor is changed in color. The bleaching is in proportion to the duration of exposure, provided the radiation is constant. The standard solution is kept in brown glass bottles but for use is contained in a silica tube. This acetone-blue solution in a layer 3 mm. thick transmits the line 3342 Å. U. of the mercury vapor lamp, partly absorbs 3132 Å. U. and transmits nothing shorter than this wave-length. After the blue solution had been acted on by radiation its color was compared with that of a series of eight tubes fixed and unbleachable in color, in which the degrees of the color scale were equal as biologically standardized through the killing of bacteria and the production of erythema on the human skin.

"Ultra-violet rays reach us diffusely from the blue sky and

bright clouds as well as direct from the sun. Dorno has shown that the ultra-violet radiation from the clear sky at Davos is greater than from the low sun. The silica tube containing the standard acetone-blue solution must then be exposed to the sky right down to the horizon line. At one station the reading appeared to be 30–40 per cent. too low. The full exposure of the tube to the sky was found to be obstructed by trees. It was agreed that the tube should be exposed in future on the top of a tall mast. The readings continued to be too low. The tube was then found to be screened from the northern half of the sky by an obstruction on the top of the mast. On raising the tube above the obstruction the true readings were obtained."

A series of determinations was made over a period of more than two years and at several places in England. The effect of the ultra-violet light was strongest in June and weakest in February and November. "Places remote from towns, such as Peppard Common and the top of Hamstead, which is singularly free from smoke during the greater part of the year owing to its height and position to the northwest of London, receive considerably more ultra-violet light than those more exposed to the smoke clouds of industrial areas. Ultra-violet intensity is clearly more dependent upon artificial air pollution than to natural causes due to geographical position in England." Davos in Switzerland gets more ultra-violet radiation than the most favored place in England but Assouan in Egypt gets more than Davos. G. F. S.

Confusion worse Confounded. It was said in the '70's of the last century in medical circles that the use of potassium chlorate in American medical practice (now fortunately almost abandoned) was due to a mistranslation from the German, the original article recommending "kalium chloratum," which is potassium chloride, but taken by the translator as meaning potassium chlorate, which latter would be in German "kalium chloricum." Whether this statement is or is not true, there are many instances in which erroneous translations have caused serious misleadings.

The prompting for this introduction is the confusion that has crept into the discussions as to the effects of baking powders using alum, generally in the form of sodium alum. With curious frequency, the parties engaged in this widespread and somewhat acrimonious dispute, talk about the effects of "alum," when there is no alum in the material after the leavening has taken place. Baking powders are always made with a slight excess of carbonate and the whole of the aluminum salt is transformed into aluminum hydroxide. In an article recently published in *Science* (1928, 68, 161)

being the current number (August 17) repeated statements are made that "alum" occurs in fruits, vegetables, and meats, whereas the statement should be that "aluminum" so occurs. There is no alum of any kind in the article served at our tables. Alum is found in the ordinary kitchen only in the unchanged baking powder. One sentence from the article will serve to show the peculiar misuse of the term. "Analyses of agricultural products made by Langworthy and Austen in the Department of Agriculture, show the quantities of alum in our agricultural food products to be extremely minute." The total amount of "alum" in these products would be correctly represented by "decimal nought."

H. L.

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